

Entry Task: Show me “step 0” of these HW problems, that is ***complete the square...***

6. Question Details

Evaluate the integral. (Use C for t

$$\int \frac{x^2}{(3 + 4x - 4x^2)^{3/2}} dx$$

7. Question Details

Evaluate the integral. (Re

$$\int \sqrt{x^2 + 6x} dx$$

8. Question Details

Evaluate the integral.

$$\int \frac{2x^2 + 6}{(x^2 - 2x + 2)^2} dx$$

7.4 Partial Fractions

Goal: Method for rational functions.

Motivation

$$\int \frac{x^3 + 4x - 4}{x^2(x^2 + 4)} dx = ??$$

We learn to break the fraction into:

$$\frac{x^3 + 4x - 4}{x^2(x^2 + 4)} = \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^2 + 4}$$

Then we integrate separately.

$$\int \frac{x^3 + 4x - 4}{x^2(x^2 + 4)} dx = \int \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^2 + 4} dx$$

Partial Fraction Decomposition

Step 0: Is the fraction *reduced*?

- If yes, move to step 1.
- If not, divide, then move to step 1.

A fraction skill:

Reduce the “improper” fraction:

$$\frac{576}{11} = ? + \frac{?}{11}$$

Is the highest power on top strictly smaller than the highest power on bottom?

Example:

$$\int \frac{x^2 + x}{x + 3} dx$$

Partial Fractions Method Summary

Step 0: Reduce (if needed).

Step 1: Factor & write decomposition.

i) *Distinct Linear:*
$$\frac{x^2-3}{x(x-1)(x+4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+4}$$

ii) *Repeated Linear:*
$$\frac{5+2x}{(x+3)(x-2)^3} = \frac{A}{x+3} + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3}$$

iii) *Irreducible Quadratic:*
$$\frac{4x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$$

Step 2: Solve for A, B, C...

Step 3: Integrate

All the integrals end up like one of:

The method uses algebra to rewrite **any** rational function as a sum of integrals like these.

$$\int \frac{1}{2x+5} dx = \frac{1}{2} \ln|2x+5| + C$$

$$\int \frac{1}{(x-4)^2} dx = -\frac{1}{x-4} + C$$

$$\int \frac{1}{(x+7)^3} dx = -\frac{1}{2} \frac{1}{(x+7)^2} + C$$

$$\int \frac{1}{x^2+9} dx = \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C$$

$$\int \frac{x}{x^2+9} dx = \frac{1}{2} \ln|x^2+9| + C$$

Example:

$$\int \frac{x + 1}{x^2 - 4} dx$$

Example:

$$\int \frac{x+1}{x^3+3x^2} dx$$

Example:

$$\int \frac{x^2 - x + 6}{x^3 + 3x} dx$$

Example:

$$\int \frac{x}{x^2 + 4x + 5} dx$$

Section 7.5: Integration Summary

How to integrate

A. Simplify and look for substitutions

B. Products/Logs/Inv. Trig → BY PARTS

Sin/Cos/Tan/Sec combos → TRIG

Quadratic (under radical) → TRIG SUB

Rational Function → PART FRAC.

C. If nothing seems to work, substitution.

($u = \text{inside}$, $u = \sqrt{\quad}$, $u = \text{trig}$, $u = e^x$)

Common Substitution Examples:

1. $\int e^{\sqrt{x}} dx$

3. $\int \frac{\cos(x)}{4 - \sin^2(x)} dx$

2. $\int \frac{3}{x - 2\sqrt{x}} dx$

4. $\int e^x \cos(e^x) \sin^3(e^x) dx$

How would you *start* these?

1. $\int \tan^3(x) \sec(x) \, dx$

2. $\int x^2 \ln(x) \, dx$

3. $\int x \sqrt{5 - x^2} \, dx$

4. $\int \frac{\sqrt{x^2 - 1}}{x^2} \, dx$

5. $\int \frac{x^2 + 1}{x^2 - 2x - 3} dx$

6. $\int x \tan^{-1}(x) dx$

7. $\int \frac{dx}{\sqrt{4x^2 + 8x - 12}} dx$