

7.3 Trigonometric Substitution

- A) Substitute, don't forget $dx = ??d\theta$.
Simplify (eliminate root)
- B) Use 7.2 methods for trig integrals.
- C) Draw a triangle and return to x .

Entry Task: Try to start evaluating

$$\int \sqrt{9 + x^2} dx$$

CASE	SUBSTITUTION
$a^2 - x^2$	$x = a \sin(\theta)$
$a^2 + x^2$	$x = a \tan(\theta)$
$x^2 - a^2$	$x = a \sec(\theta)$

Example:

$$\int \frac{\sqrt{x^2 - 16}}{x} dx$$

Most important application:

Area under a circle

$$\int \sqrt{4 - x^2} dx$$

Completing the Square:

$$\sqrt{ax^2 + \mathbf{bx} + c}$$

If you encounter a “**middle term**”,
then complete the square.

Example: $\sqrt{4x^2 - 32x + 100}$

i) **Factor out the “a”.**

$$\sqrt{4(x^2 - 8x + 25)} = 2\sqrt{x^2 - 8x + 25}$$

ii) **Add/subtract “half-middle squared”**

$$\text{Half of middle} = (-8)/2 = -4$$

$$\text{Squared} = (-4)^2 = 16$$

$$2\sqrt{x^2 - 8x + 16 - 16 + 25}$$

iii) **Factor the perfect square**

$$2\sqrt{(x - 4)^2 + 25}$$

iv) **Check your work!!!!**

Example:

$$\int \frac{1}{\sqrt{4x^2 - 32x + 100}} dx$$

Example:

$$\int \frac{x}{\sqrt{16 - 6x - x^2}} dx$$