7.2: Trig Integrals Review - Dr. Loveless

1. **SINES AND COSINES**
   (a) If $\sin(x)$ or $\cos(x)$ is to an odd power. Factor out a term from the odd power. Use identity $\sin^2(x)+\cos^2(x)=1$. Then substitute.
   (b) If $\sin(x)$ and $\cos(x)$ both with even powers: Use $\sin^2(x)=\frac{1}{2}(1-\cos(2x))$, $\cos^2(x)=\frac{1}{2}(1+\cos(2x))$.

2. **TANGENTS AND SECANTS**
   (a) If $\sec(x)$ has an even power. Factor out $\sec^2(x)$. Use identity $\sec^2(x)=\tan^2(x)+1$. Then substitute.
   (b) If $\tan(x)$ has an odd power (and at least one $\sec(x)$). Factor out $\sec(x)\tan(x)$. Use identity $\tan^2(x)=\sec^2(x)-1$. Then substitute.

3. **EXAMPLES**
   - *Odd sine.* $\displaystyle\int \sin^3(x)\cos^2(x)\,dx=\int \sin^2(x)\cos^2(x)\sin(x)\,dx=\int (1-\cos^2(x))\cos^2(x)\sin(x)\,dx.$
   - *Odd cosine.* $\displaystyle\int \sin^4(x)\cos^3(x)\,dx=\int \sin^4(x)\cos^2(x)\cos(x)\,dx=\int \sin^4(x)(1-\sin^2(x))\cos(x)\,dx.$
   - *Only even powers.* $\displaystyle\int \sin^2(x)\,dx=\int \frac{1}{2}(1-\cos(2x))\,dx.$
   - *Only even powers.* $\displaystyle\int \cos^4(x)\,dx=\int \left(\frac{1}{2}(1+\cos(2x))\right)^2\,dx=\frac{1}{4}\int \left(1+2\cos(2x)+\cos^2(2x)\right)\,dx.$
     Now use half-angle on $\cos^2(2x)=\frac{1}{2}(1+\cos(4x))$.
   - *Even secant.* $\displaystyle\int \tan^2(x)\sec^4(x)\,dx=\int \tan^2(x)\sec^2(x)\sec^2(x)\,dx=\int \tan^2(x)(\tan^2(x)+1)\sec^2(x)\,dx.$
   - *Odd tangent.* $\displaystyle\int \tan^3(x)\sec^2(x)\,dx=\int \tan^2(x)\sec(x)\sec(x)\tan(x)\,dx=\int (\sec^2(x)-1)\sec(x)\sec(x)\tan(x)\,dx.$

4. **NOTES**
   (a) For $\cot(x)/\csc(x)$ the cases would be nearly identical to $\tan(x)/\sec(x)$.
   (b) If you are stuck, try changing everything to $\sin(x)$ and $\cos(x)$ (or changing everything to $\sec(x)$ and $\tan(x)$). If you are still stuck, look at all your trig identities and rewrite the integral in another way.
   (c) And **remember** that we have added the following to our table of known integrals (use these, you don't have to derive them):
   $$\int \tan(x)\,dx=\ln|\sec(x)|+C \text{ (in 5.5)}$$
   $$\int \sec(x)\,dx=\ln|\sec(x)+\tan(x)|+C \text{ (in 7.2)}$$
   $$\int \sec^3(x)\,dx=\frac{1}{2}\left(\sec(x)\tan(x)+\ln|\sec(x)+\tan(x)|\right)+C \text{ (in 7.2)}$$
   (d) *Side-note on integrating waves of different frequencies:* We won't integrate waves with different frequencies in this course, but if you ever have to, these product identities are very useful:
   $$\sin(A)\cos(B)=\frac{1}{2}(\sin(A-B)+\sin(A+B))$$
   $$\sin(A)\sin(B)=\frac{1}{2}(\cos(A-B)-\cos(A+B))$$
   $$\cos(A)\cos(B)=\frac{1}{2}(\cos(A-B)+\cos(A+B))$$
   For example: $\displaystyle\int \sin(3x)\cos(2x)\,dx=\int \frac{1}{2}\left(\sin(x)+\sin(5x)\right)\,dx.$