

7.1 Integration by Parts (*continued*)

Entry Task: Evaluate

$$\int \tan^{-1}(x) dx = \int \arctan(x) dx$$

Summary: $\int u dv = uv - \int v du$

1. Pick $u = ??$. The rest is dv .
2. Compute du and v .

Integration by parts is good for:

Products: $x e^x, x^2 \cos(3x), x \sin(5x)$

Logs: $\ln(x), x^{10} \ln(x), \frac{\ln(x)}{x^3}, \dots$

Inv. Tri: $\sin^{-1}(x), x \tan^{-1}(x), \dots$

Products: $e^x \sin(x), e^x \cos(x)$

7.2 Trigonometric Integral Methods

Goal: A procedure to integrate *any* combination of trig functions.

Motivating examples:

These are substitution problems, what is u ?

$$\int \sin^3(x) (1 - \sin^2(x)) \cos(x) dx$$

$$\int (1 - \cos^2(x)) \cos^5(x) \sin(x) dx$$

$$\int \tan^5(x) (1 + \tan^2(x)) \sec^2(x) dx$$

$$\int \sec^6(x) \sec(x) \tan(x) dx$$

7.2 idea: Use trig identities to turn almost all trig problems into one of these situations!

Case 1

(cos(x) or sin(x) has odd power)

Special tool needed:

$$\sin^2(x) + \cos^2(x) = 1$$

i) $\int \sin^2(x) \cos^3(x) dx$

ii) $\int \sin^3(x) dx$

Case 2

(both $\sin(x)$, $\cos(x)$ even powers)

$$i) \int \cos^2(x) dx$$

Special tools needed:

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$\sin(x) \cos(x) = \frac{1}{2} \sin(2x)$$

$$ii) \int \sin^4(x) dx$$

Case 3 (even power on $\sec(x)$)

$$\int \tan^2(x) \sec^4(x) dx$$

Special tool needed:

$$\tan^2(x) + 1 = \sec^2(x)$$

**Case 4 (odd power on $\tan(x)$, and
at least one $\sec(x)$)**

$$\int \tan^3(x) \sec^5(x) dx$$

Special tool needed:

$$\sec^2(x) - 1 = \tan^2(x)$$

Notes: If integral doesn't fit...

1. Rewrite as $\sin(x)$ & $\cos(x)$.
2. Rewrite as $\sec(x)$ & $\tan(x)$.
3. Try using trig identities.

And there are still a few “holes”.

Particularly, odd power on $\sec(x)$.

For these you can quote

(proof in the book):

$$\int \tan(x) \, dx = \ln|\sec(x)| + C$$

$$\int \sec(x) \, dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int \sec^3(x) \, dx = \frac{1}{2} \sec(x) \tan(x) + \frac{1}{2} \ln|\sec(x) + \tan(x)| + C$$

What is the first step in each?

$$\int \sin^3(x)\cos^4(x)dx$$

$$\int \sin^5(x)\cos^3(x)dx$$

$$\int \cos^4(x)dx$$

$$\int \tan^5(x)\sec^4(x)dx$$

$$\int \tan^5(x)\sec(x)dx$$