Goal: Finish learning how to integrate trig function and introduce trig substitution.

7.2 Trig Integrals (*continued*)

Entry Task: Fill in the blanks in the tables of identities, then try to integrate

$$\int \sin^3(\mathrm{x})\cos^2(\mathrm{x})dx$$

Square Identities
$\sin^2(x) = 1 -$
$\cos^2(x) = 1 -$
$\tan^2(x) =$
$\sec^2(x) =$

Half Angle Identities
$\sin^2(x) = \frac{1}{2}$
$\cos^2(x) = \frac{1}{2}$
$\sin(x)\cos(x) = \frac{1}{2}$

Case 1

cos(x) or sin(x) with odd power

1. Factor out one from an odd power.
2. Change rest of integrand to other function via
   $\sin^2(x) = 1 - \cos^2(x)$ or $\cos^2(x) = 1 - \sin^2(x)$
3. Use u-substitution.

Example

$$\int \sin^3(\mathrm{x})\cos^5(\mathrm{x})dx$$

Case 2

Both sin(x), cos(x) even powers

1. Factor into squares.
2. Use $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ or $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
3. Expand and simplify
   Use half-angle again if you encounter another square.
4. Continue until done!

Example:

$$\int \cos^4(x)\, dx$$

Case 3

sec*(x) has an even power*

1. Factor out $\sec^2(x)$.
2. Change rest of integrand to $\tan(x)$ via
$$\sec^2(x) = 1 + \tan^2(x)$$
3. Use u-substitution.

Example:

$$\int \tan^5(\mathrm{x})\sec^4(\mathrm{x})dx$$

Case 4

tan*(x) has an odd power and at least one sec(x)*

1. Factor out $\sec(x)\tan(x)$.
2. Change rest of integrand to $\sec(x)$ via $\tan^2(x) = \sec^2(x) - 1$
3. Use u-substitution.

Example:

$$\int \tan^3(\mathrm{x})\sec(\mathrm{x})dx$$

Tried the four cases and still stuck?

Try:

1. Rewrite in terms of sin(x) and cos(x).
2. Rewrite in terms of sec(x) and tan(x).
3. Try using trig identities.

There are still a few “holes”.

Namely, when there is one tan(x) or an odd power on sec(x). You can quote these (proof in the book):

$$\int \tan(x)\,dx = \ln|\sec(x)| + C$$

$$\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int \sec^3(x)\,dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + C$$

7.3 Trigonometric Substitution

Goal: Evaluate integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{a^2 + x^2}$, or $\sqrt{x^2 - a^2}$.

CASE	SUBSTITUTION
$a^2 - x^2$	$x = a\sin(\theta)$, $-\pi/2 \le \theta \le \pi/2$
$a^2 + x^2$	$x = a\tan(\theta)$, $-\pi/2 < \theta < \pi/2$
$x^2 - a^2$	$x = a\sec(\theta)$, $0 \le \theta < \pi/2$, $\pi \le \theta < 3\pi/2$

Example:

1. $\int \frac{x^3}{\sqrt{4 - x^2}}\, dx$

Trigonometric Substitution Method:

A) Substitute, don't forget dx = ??dθ.
   Simplify (eliminate root)
B) Use 7.2 methods for trig integrals.
C) Draw a triangle and return to x.

2. $\int \sqrt{9 + x^2}\,dx$

3. $\int \frac{\sqrt{x^2 - 16}}{x} dx$