7.1 Integration by Parts

Goal: Reverse the product rule.

Before we start, add to your list:

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{1}{\mathrm{ax+b}}\,dx = \frac{1}{a}\ln|ax+b| + C$$

Derivation of Integration By Parts

The product rule says:

$$u(x)v'(x) + v(x)u'(x) = \frac{d}{dx}\big(u(x)v(x)\big)$$

which can be written as

$$\int u(x)v'(x)dx + \int v(x)u'(x)dx = u(x)v(x)$$

Writing in terms of the differentials

$$dv = v'(x)dx \ \text{ and } \ du = u'(x)dx$$

we have

$$\int u\,dv + \int v\,du = uv$$

which we rearrange to get

Integration by Parts formula:

$$\int u\,dv = uv - \int v\,du$$

Example:

$$\int x \cos(8x) dx$$

Step 1: Choose *u* and *dv*.
Step 2: Compute *du* and *v*.
Step 3: Use formula (and hope)

Example:

$$\int x^2 \ln(x)\, dx$$

Follow-up:

$$\int_1^e x^2 \ln(x)\, dx$$

Notes:

1. Symbols *u* and *v* **never** appear integral.
   Just locations in the formula.
   (no variables are changing).

2. *u* and *dv* completely split up integrand.
   Choose *u*, then *dv* is everything else.

3. The goal: Make

$$\int v\,du \quad \text{nicer than} \quad \int u\,dv$$

   (a) u = “has a simple derivative”

   (b) dv = “something you can integrate”

   (c) *Hopefully* “v du” is easier to integrate!

Example:

$$\int \sin^{-1}(x)\, dx$$

Example:

$$\int x^2 e^{\frac{x}{2}} dx$$

Example:

$$\int e^x \cos(x)\, dx$$