1 $\displaystyle\int \frac{y^3-4y+7}{y^2+2y-3}\,dy$

FIRST DIVIDE

THEN PARTIAL FRACTIONS

$$\boxed{\frac{1}{2}y^2-2y+\ln|y-1|+2\ln|y+3|+C}$$

2 $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}\,dx$

U-SUBSTITUTION $\underline{\text{OR}}$ TRIG. SUBSTITUTION

$u=x^2+4$ $\qquad$ $x=2\tan(\theta)$

$$\boxed{\frac{1}{3}x^2\sqrt{x^2+4}-\frac{8}{3}\sqrt{x^2+4}+C}$$

SAME AS $\frac{1}{3}(x^2+4)^{3/2}-4(x^2+4)^{1/2}+C$

3 $\displaystyle\int \sin^2(t)\cos^2(t)\,dt$

EVEN POWERS $\Rightarrow$ HALF-ANGLE

(ONCE ON $\sin^2(t)$, ONCE ON $\cos^2(t)$

AND ONE MORE TIME AFTER THAT)

$$\boxed{\frac{1}{8}t-\frac{1}{32}\sin(4t)+C}$$

4 $\displaystyle\int \sin^3(x)\tan^2(x)\cos^2(x)\,dx$

MAKE ALL INTO sin/cos

$\Rightarrow \int \sin^5(x)\,dx$

ODD POWER $\Rightarrow \int (1-\cos^2(x))^2\sin(x)\,dx$

U-SUB.

$$\boxed{-\cos(x)+\frac{2}{3}\cos^3(x)-\frac{1}{5}\cos^5(x)+C}$$

5 $\displaystyle\int \frac{dx}{\sqrt{4x^2+8x-12}}$

COMPLETE SQUARE

$\Rightarrow \sqrt{4(x^2+2x-3)} = 2\sqrt{x^2+2x+1-1-3}$

$= 2\sqrt{(x+1)^2-4}$

TRIG SUB. $\Rightarrow$ $x+1=2\sec\theta$

$$\boxed{\frac{1}{2}\ln\left|\frac{x+1}{2}+\frac{\sqrt{x^2+2x-3}}{2}\right|+C}$$

6 $\displaystyle\int x\tan^{-1}(x)\,dx$

BY PARTS.

$u=\tan^{-1}(x)$ $\quad$ $dv=x\,dx$

$$\boxed{\frac{1}{2}x^2\tan^{-1}(x)-\frac{1}{2}x+\frac{1}{2}\tan^{-1}(x)+C}$$

7 $\displaystyle\int xe^{3x+1}\,dx$

BY PARTS

$u=x$ $\qquad$ $dv=e^{3x+1}dx$

$$\boxed{\frac{1}{3}xe^{3x+1}-\frac{1}{9}e^{3x+1}+C}$$

8 $\displaystyle\int \frac{x^3+2}{x^2-1}\,dx$

DIVIDE, FIRST

THEN PARTIAL FRACTIONS

$$\boxed{\frac{1}{2}x^2+\frac{3}{2}\ln|x-1|-\frac{1}{2}\ln|x+1|+C}$$

9 $\displaystyle\int \frac{x^3}{x^2+1}\,dx$

DIVIDE, FIRST

$$\begin{array}{r} x \\ x^2+1\,\overline{)\,x^3} \\ -(x^3+x) \\ \hline -x \end{array}$$

$\int x - \underbrace{\frac{x}{x^2+1}}_{\text{SUBSTITUTION}}\,dx$

$$\boxed{\frac{1}{2}x^2-\frac{1}{2}\ln|x^2+1|+C}$$

10 $\displaystyle\int \frac{1}{x^2\sqrt{x^2-1}}\,dx$

TRIG SUBSTITUTION. $x=\sec(\theta)$

$$\boxed{\frac{\sqrt{x^2-1}}{x}+C}$$

11 $\int \frac{4}{x^2(x+2)}\,dx$

PARTIAL FRACTIONS

$$4\left(\boxed{-\frac{1}{4}\ln|x| - \frac{1}{2}\frac{1}{x} + \frac{1}{4}\ln|x+2|}\right) + C$$

12 $\int \frac{x^3}{x^2-4}\,dx$

DIVIDE, THEN PARTIAL FRACTIONS

$$\boxed{\frac{1}{2}x^2 + 2\ln|x+2| + 2\ln|x-2| + C}$$

13 $\int \frac{x^3}{\sqrt{9-x^2}}\,dx$

U-SUB OR TRIG. SUB.

$u = 9-x^2$ $\qquad x = 3\sin(\theta)$

$$\boxed{-\frac{1}{3}x^2\sqrt{9-x^2} - 6\sqrt{9-x^2} + C}$$

SAME AS $-9(9-x^2)^{1/2} + \frac{1}{3}(9-x^2)^{3/2} + C$

14 $\int \frac{3}{x-2\sqrt{x}}\,dx$

U-SUB. $u = \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}dx \Rightarrow dx = 2\sqrt{x}\,du = 2u\,du$

$x = u^2$

$$\int \frac{6u}{u^2-2u}\,du = \int \frac{6}{u-2}\,du$$

$$\boxed{6\ln|\sqrt{x}-2| + C}$$

15 $\int \frac{\cos(x)}{4-\sin^2(x)}\,dx$

U-SUB. $u = \sin(x) \Rightarrow \int \frac{1}{4-u^2}\,du$

THEN PARTIAL FRACTIONS

$$\boxed{\frac{1}{4}\ln|\sin(x)+2| - \frac{1}{4}\ln|\sin(x)-2| + C}$$

16 $\int \frac{e^{1/x}}{x^2}\,dx$

U-SUB. $u = \frac{1}{x}$ $\quad du = -\frac{1}{x^2}dx$

$-\int e^u\,du = -e^u + C$

$$\boxed{-e^{1/x} + C}$$

17 $\int \frac{x}{x^2+2x+5}\,dx$

IRREDUCIBLE $\Rightarrow$ COMPLETE THE SQUARE

$x^2+2x+5 = x^2+2x+1-1+5 = (x+1)^2+4$

$t = x+1$, $dt = dx \Rightarrow \int \frac{t-1}{t^2+4}\,dt$

SPLIT INTO TWO.

$$\boxed{\frac{1}{2}\ln|x^2+2x+5| - \frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C}$$

18 $\int \frac{1}{(9-x^2)^{3/2}}\,dx$

TRIG. SUBSTITUTION. $x = 3\sin(\theta)$

$$\boxed{\frac{1}{9}\frac{x}{\sqrt{9-x^2}} + C}$$

19 $\int \cos^6(3x)\sec^3(3x)\,dx$

REWRITE AS ALL COSINES

$\int \cos^3(3x)\,dx \Rightarrow$ ODD COSINE

$\int (1-\sin^2(3x))\cos(3x)\,dx$ $\qquad u = \sin(3x)$, $du = 3\cos(3x)\,dx$

$\frac{1}{3}\int 1-u^2\,du$

$$\boxed{\frac{1}{3}\sin(3x) - \frac{1}{9}\sin^3(3x) + C}$$

20 $\int x^5\ln(x)\,dx$

BY PARTS $\quad u = \ln(x) \quad dv = x^5dx$

$$\boxed{\frac{1}{6}x^6\ln|x| - \frac{1}{36}x^6 + C}$$

21 $\int \sin^3(x)\cos^6(x)\,dx$

ODD SINE $\Rightarrow \int (1-\cos^2(x))\cos^6(x)\sin(x)\,dx$

$u = \cos(x)$

$$\boxed{-\frac{1}{7}\cos^7(x) + \frac{1}{9}\cos^9(x) + C}$$

22 $\int \frac{5}{x^3+2x}\,dx$

PARTIAL FACTIONS

$$\boxed{\frac{5}{2}\ln|x| - \frac{5}{4}\ln|x^2+2| + C}$$

23 $\int \frac{dx}{x^2-8x+18}$

IRREDUCIBLE $\Rightarrow$ COMPLETE THE SQUARE

$x^2-8x+18 = x^2-8x+16-16+18 = (x-4)^2+2$

$u = x-4 \quad du = dx.$

$$\boxed{\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-4}{\sqrt{2}}\right) + C}$$

24 $\int \frac{dx}{x^3+3x^2}$

PARTIAL FRACTIONS

$$\boxed{-\frac{1}{9}\ln|x| - \frac{1}{3}\frac{1}{x} + \frac{1}{9}\ln|x+3| + C}$$

25 $\int \frac{e^{3x}}{e^{2x}-1}\,dx$

U-SUB. $u = e^x,\ du = e^x dx$

$\int \frac{u^2}{u^2-1}\,du \Rightarrow$ DIVIDE & PARTIAL FRACTIONS

$$\boxed{e^x + \frac{1}{2}\ln|e^x-1| - \frac{1}{2}\ln|e^x+1| + C}$$

26 $\int \frac{t^3}{\sqrt{t^2+4}}\,dt$ $\quad u = t^2+4$ OR $t = 2\tan(\theta)$

U-SUB OR TRIG SUB.

(SAME PROBLEM AS 2)

$$\boxed{\frac{1}{3}t^2\sqrt{t^2+4} - \frac{8}{3}\sqrt{t^2+4} + C}$$

27 $\int 4y\sec^2(2y)\,dy$

BY PARTS.

$u = 4y \qquad dv = \sec^2(2y)\,du \qquad t = 2y$

$du = 4dy \qquad v = \frac{1}{2}\tan(2y) \qquad dt = 2dy$

$dy = \frac{1}{2}dt$

$$\boxed{2y\tan(2y) - \ln|\sec(2y)| + C}$$

28 $\int_0^{x} x\sqrt{1-x}\,dx$

U-SUB. $\quad u = 1-x \Rightarrow x = 1-u$

$du = dx$

$\int (1-u)\sqrt{u}\,du = \int u^{1/2} - u^{3/2}\,du$

$$\boxed{\frac{2}{3}(1-x)^{3/2} - \frac{2}{5}(1-x)^{5/2} + C}$$

29 $\int \frac{x+1}{\sqrt{5+4x-x^2}}\,dx$

COMPLETE THE SQUARE $\Rightarrow$ TRIG SUB.

$5+4x-x^2 = 5+4-4+4x-x^2 = 9-(x-2)^2$

$x-2 = 3\sin(\theta)$

$$\boxed{-\sqrt{5+4x-x^2} + 3\sin^{-1}\left(\frac{x-2}{3}\right) + C}$$

30 $\int \frac{2}{x^3-3x^2+x-3}\,dx$

PARTIAL FRACTIONS

$x^3-3x^2+x-3 = x^2(x-3)+(x-3)$

$= (x-3)(x^2+1)$

$$\boxed{\frac{1}{10}\ln|x-3| - \frac{1}{20}\ln|x^2+1| - \frac{3}{10}\tan^{-1}(x) + C}$$