① **Partial Fractions**

$$\frac{x+2}{x^2+5x} = \frac{x+2}{x(x+5)} = \frac{A}{x} + \frac{B}{x+5}$$

$$x+2 = A(x+5) + Bx$$

$x=0 \Rightarrow 2 = A(5) \quad A = 2/5$

$x=-5 \Rightarrow -3 = B(-5) \quad B = 3/5$

$$\int \frac{x+2}{x^2+5x}\,dx = \int \frac{2/5}{x} + \frac{3/5}{x+5}\,dx$$

$$= \frac{2}{5}\int \frac{1}{x}\,dx + \frac{3}{5}\int \frac{1}{x+5}\,dx$$

$$= \boxed{\frac{2}{5}\ln|x| + \frac{3}{5}\ln|x+5| + C}$$

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② **TRIG** ODD COS

$\int \sin^{10}(x)\cos^2(x)\cos(x)\,dx$ $\quad u = \sin(x)$

$\int \sin^{10}(x)(1-\sin^2(x))\cos(x)\,dx$ $\quad du = \cos(x)\,dx$

$\int u^{10}(1-u^2)\,du$

$\int u^{10} - u^{12}\,du = \frac{1}{11}u^{11} - \frac{1}{13}u^{13} + C$

$= \boxed{\frac{1}{11}\sin^{11}(x) - \frac{1}{13}\sin^{13}(x) + C}$

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③ **By Parts** log & product

$u = \ln(x) \quad dv = x^6\,dx$

$du = \frac{1}{x}dx \quad v = \frac{1}{7}x^7$

$\frac{1}{7}x^7\ln(x) - \frac{1}{7}\int x^7 \frac{1}{x}\,dx$

$\frac{1}{7}x^7\ln(x) - \frac{1}{7}\int x^6\,dx$

$\boxed{\frac{1}{7}x^7\ln(x) - \frac{1}{49}x^7 + C}$

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④ **Partial Fractions (Division)**

$$\begin{array}{r} x^2 - x + 1 \\ x+1 \,\overline{)\, x^3} \\ -(x^3 + x^2) \\ \hline -x^2 \\ -(-x^2 - x) \\ \hline x \\ -(x+1) \\ \hline -1 \end{array}$$

$\int \frac{x^3}{x+1}\,dx$

$= \int x^2 - x + 1 - \frac{1}{x+1}\,dx$

$= \boxed{\frac{1}{3}x^3 - \frac{1}{2}x^2 + x - \ln|x+1| + C}$

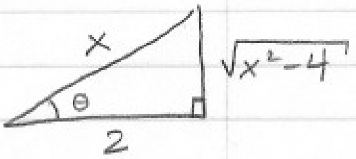
⑤ $\boxed{\text{Trig Sub}}$

$$x = 2\sec(\theta)$$
$$dx = 2\sec(\theta)\tan(\theta)\,d\theta$$
$$\sqrt{x^2-4} = \sqrt{4\sec^2(\theta) - 4} = 2\tan(\theta)$$

$$\int \frac{1}{\sqrt{x^2-4}}\,dx = \int \frac{1}{\cancel{2\tan(\theta)}}\,2\sec(\theta)\cancel{\tan(\theta)}\,d\theta$$
$$= \int \sec(\theta)\,d\theta$$
$$= \ln|\sec(\theta) + \tan(\theta)| + C$$
$$= \boxed{\ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right| + C}$$

$\boxed{\sec(\theta) = \frac{x}{2}}$

$\frac{hyp}{adj}$



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⑥ $\boxed{\text{Partial Fractions}}$

$$\frac{1}{x^3+x} = \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$
$$\Rightarrow \quad 1 = A(x^2+1) + (Bx+C)x$$

$x = 0 \Rightarrow 1 = A(1)$ $\quad \boxed{A=1}$

$x = 1 \Rightarrow 1 = 2 + (B+C)$

$\boxed{-1 = B+C}$

$x = -1 \Rightarrow 1 = 2 - (-B+C)$

$\boxed{-1 = B - C}$ $\quad B = C-1$

alternate method

$$\begin{cases} 1 = x^2+1 + Bx^2 + Cx \\ 0 = (1+B)x^2 + Cx \\ \text{so } C = 0 \text{ or } 1+B=0 \end{cases}$$

$-1 = C-1+C$

$0 = 2C$

$\boxed{C=0}$

$\boxed{B = C-1 = -1}$

$$\int \frac{dx}{x^3+x} = \int \frac{1}{x} + \frac{-x+0}{x^2+1}\,dx$$
$$= \ln|x| - \int \frac{x}{x^2+1}\,dx$$
$$= \ln|x| - \int \frac{1}{u}\frac{1}{2}\,du$$
$$= \ln|x| - \frac{1}{2}\ln|u| + C$$
$$= \boxed{\ln|x| - \frac{1}{2}\ln|x^2+1| + C}$$

$u = x^2+1$

$du = 2x\,dx \qquad dx = \frac{1}{2x}\,du$

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⑦ $\boxed{\text{By Parts}}$

$u = x \qquad dv = \sec(x)\tan(x)\,dx$

$du = dx \qquad v = \sec(x)$

$$x\sec(x) - \int \sec(x)\,dx$$
$$\boxed{x\sec(x) - \ln|\sec(x) + \tan(x)| + C}$$

(8) $\boxed{\text{u-substitution}}$

$u = \tan^{-1}(x)$

$du = \frac{1}{x^2+1}dx$

$dx = x^2+1\, du$

$$\int \frac{\sqrt{u}}{\cancel{x^2+1}} \cancel{(x^2+1)}\, du$$
$$= \int u^{1/2}\, du$$
$$= \frac{2}{3}u^{3/2} + C = \boxed{\frac{2}{3}\left(\tan^{-1}(x)\right)^{3/2} + C}$$

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(9) $\boxed{\text{Try identities / simplifying first}}$

$\sin^2(t) = \frac{1}{2}(1-\cos(2t))$

$$\int \frac{dt}{\sin^2(t)+\cos(2t)} = \int \frac{dt}{\frac{1}{2}-\frac{1}{2}\cos(2t)+\cos(2t)}$$
$$= \int \frac{dt}{\frac{1}{2}+\frac{1}{2}\cos(2t)}$$
$$= \int \frac{dt}{\cos^2(t)} = \int \sec^2(t)\,dt$$
$$= \boxed{\tan(t) + C}$$

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(10) $\boxed{\text{Trig Sub}}$

$x = 2\sin(\theta)$

$dx = 2\cos(\theta)d\theta$

$4-x^2 = 4-4\sin^2(\theta)$

$= 4\cos^2(\theta)$

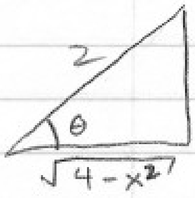
$$\int \frac{(2\sin(\theta))^2}{(4\cos^2(\theta))^{3/2}} 2\cos(\theta)d\theta$$

$$\int \frac{4\sin^2(\theta)}{(2\cos(\theta))^3} 2\cos(\theta)d\theta$$

$$\int \frac{4\sin^2(\theta)}{(2\cos(\theta))^2}d\theta = \int \frac{4\sin^2(\theta)}{4\cos^2(\theta)}d\theta = \int \tan^2(\theta)d\theta$$

$\sin(\theta) = \frac{x}{2}$

$$= \int \sec^2(\theta) - 1\, d\theta$$
$$= \tan(\theta) - \theta + C$$
$$= \boxed{\frac{x}{\sqrt{4-x^2}} - \sin^{-1}\left(\frac{x}{2}\right) + C}$$



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(11) $\boxed{\text{Simplify / Try Sub}}$

$u = e^x$

$du = e^x\,dx$

$dx = \frac{1}{e^x}du$

$$\int \frac{dx}{1+e^x} = \int \frac{1}{1+u}\frac{1}{e^x}du$$
$$= \int \frac{1}{(1+u)u}du$$

$$\frac{1}{(1+u)u} = \frac{A}{u+1} + \frac{B}{u}$$

$1 = Au + B(u+1)$

$u=0 \Rightarrow B=1 \qquad u=-1 \Rightarrow A=-1$

$$\int \frac{-1}{u+1} + \frac{1}{u}\,du = -\ln|u+1| + \ln|u| + C$$
$$= \boxed{-\ln|e^x+1| + \ln|e^x| + C}$$