7.2: TRIG INTEGRALS SUMMARY

1. SINES AND COSINES

   (a) If $\sin(x)$ or $\cos(x)$ is to an odd power.

      i. Factor out a term from the odd power. Use the identity $\sin^2(x) + \cos^2(x) = 1$.

      ii. Do a substitution ($u = \sin(x)$ or $u = \cos(x)$ as appropriate).

   (b) If $\sin(x)$ and $\cos(x)$ both have even powers: Simplify with half-angle identities.

2. TANGENTS AND SECANTS

   (a) If $\sec(x)$ has an even power.

      i. Factor out $\sec^2(x)$. Use the identity $\sec^2(x) = \tan^2(x) + 1$.

      ii. Do a substitution ($u = \tan(x)$).

   (b) If $\tan(x)$ has an odd power (and at least one $\sec(x)$):

      i. Factor out $\sec(x)\tan(x)$. Use the identity $\tan^2(x) = \sec^2(x) - 1$.

      ii. Do a substitution ($u = \sec(x)$).

   Examples:

   - Odd power on sine: Use $u = \cos(x)$.
     $\displaystyle\int \sin^3(x)\cos^2(x)\,dx = \int \sin^2(x)\cos^2(x)\sin(x)\,dx = \int (1-\cos^2(x))\cos^2(x)\sin(x)\,dx.$
   - Odd power on cosine: Use $u = \sin(x)$.
     $\displaystyle\int \sin^4(x)\cos^3(x)\,dx = \int \sin^4(x)\cos^2(x)\cos(x)\,dx = \int \sin^4(x)(1-\sin^2(x))\cos(x)\,dx.$
   - Only even powers: Integrate directly as follows:
     $\displaystyle\int \sin^2(x)\,dx = \int \frac{1}{2}(1-\cos(2x))\,dx.$
   - Only even powers:
     $\displaystyle\int \cos^4(x)\,dx = \int (\cos^2(x))^2\,dx = \int \left(\frac{1}{2}(1+\cos(2x))\right)^2\,dx = \frac{1}{4}\int 1 + 2\cos(2x) + \cos^2(2x)\,dx.$
     Now use half-angle on $\cos^2(2x) = \frac{1}{2}(1+\cos(4x))$, then integrate directly.
   - Even power on secant: Use $u = \tan(x)$.
     $\displaystyle\int \tan^2(x)\sec^4(x)\,dx = \int \tan^2(x)\sec^2(x)\sec^2(x)\,dx = \int \tan^2(x)(\tan^2(x)+1)\sec^2(x)\,dx.$
   - Odd power on tangent: Use $u = \sec(x)$.
     $\displaystyle\int \tan^3(x)\sec^2(x)\,dx = \int \tan^2(x)\sec(x)\sec(x)\tan(x)\,dx = \int (\sec^2(x)-1)\sec(x)\sec(x)\tan(x)\,dx.$

3. NOTES

   (a) For $\cot(x)/\csc(x)$ the cases would be nearly identical to $\tan(x)/\sec(x)$.

   (b) If you are stuck, try changing everything to $\sin(x)$ and $\cos(x)$ (or changing everything to $\sec(x)$ and $\tan(x)$. If you are still stuck, look at all your trig identities and rewrite the integral in another way.

   (c) And **remember** that we have added the following to our table of known integrals (use these, you don't have to derive them):
   $\int \tan(x)\ dx = \ln|\sec(x)| + C$ (in 5.5)
   $\int \sec(x)\ dx = \ln|\sec(x) + \tan(x)| + C$ (in 7.2)
   $\int \sec^3(x)\ dx = \frac{1}{2}(\sec(x)\tan(x) + \ln|\sec(x)+\tan(x)|) + C$ (in 7.2)