5.1: Steps for Reimann Approximation

Given a function, $y = f(x)$, we approximate the area 'under' the graph from $x = a$ to $x = b$ as follows:

1. Pick some positive integer n.
   $n =$ 'number of approximating rectangles'.
   Compute $\boxed{\Delta x = \frac{b-a}{n} = \text{'the width'.}}$
   Label the tick-marks:
   $x_0 = a, x_1 = a + \Delta x, x_2 = a + 2\Delta x, \ldots, x_n = b.$
   Note the pattern: $\boxed{x_i = a + i\Delta x}$.

2. Choose the point x_i^* to determine the height of each rectangle.
   Right-endpoint method: $x_i^* = x_i$.
   Left-endpoint method: $x_i^* = x_{i-1}$.
   Midpoint method: $x_i^* = \bar{x}_i = \frac{x_i + x_{i-1}}{2}$.
   In all cases, we compute the area of each rectangle by: $\boxed{\text{'Area of one rectangle'} = f(x_i^*)\Delta x}$

3. Add up all the areas ($i = 1$ to $i = n$, where i represents the i'th rectangle).

   In general, this looks like $\boxed{\sum_{i=1}^{n} f(x_i^*)\Delta x}$.

   Here is what it looks like with each method:

   $$R_n = \sum_{i=1}^{n} f(x_i)\Delta x = f(x_1)\Delta x + \cdots + f(x_n)\Delta x.$$

   $$L_n = \sum_{i=1}^{n} f(x_{i-1})\Delta x = f(x_0)\Delta x + \cdots + f(x_{n-1})\Delta x.$$

   $$M_n = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x = f(\bar{x}_1)\Delta x + \cdots + f(\bar{x}_n)\Delta x.$$

4. We then define the exact 'area' of the region to be the limit as $n \to \infty$ (if the area is defined, it doesn't matter which method you are using, they all approach the same value).

   $$\boxed{\text{`Exact Area Definition'} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x}$$