**Math 124 Finding Tangent Lines**

Here are five standard problems involving finding tangent lines. Try them out. The last two require some set up and algebra to solve. Solutions are on the following pages.

1. Find the equation for the tangent line to $y = x^3 + \sqrt{x}$ at $x = 4$.

2. Find the equation for the tangent line to $y = 7e^x + 3$ at $x = 0$.

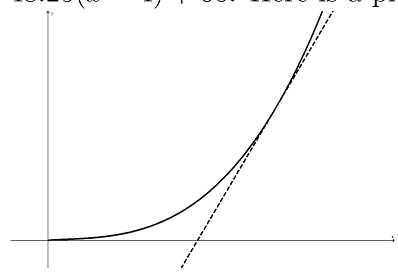
3. Find the equation for the tangent line to $y = \frac{4}{x^2} + (x+1)^2$ at $x = 1$.

4. Find the equations of the two lines that are tangent to $y = x^2$ and also pass through (0,-6).

5. Find the equation of the line that is tangent to $y = x^3$ and also pass through (0, 10).

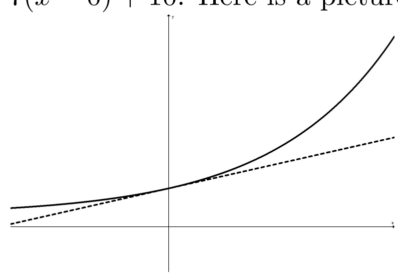
1. Find the equation for the tangent line to $y = x^3 + \sqrt{x}$ at $x = 4$.

   *SOLUTION:* At $x = 4$, we have $y(4) = 4^3 + \sqrt{4} = 66$. Thus, the line goes through the point $(4, 66)$ and is of the form $y = m(x - 4) + 66$.
   Next, $y' = 3x^2 + \frac{1}{2}x^{-1/2} = 3x^2 + \frac{1}{2\sqrt{x}}$. Thus, the slope at $x = 4$ is $y'(4) = 3(4)^2 + \frac{1}{2\sqrt{4}} = 48.25$.
   Therefore, the tangent line is $y = 48.25(x - 4) + 66$. Here is a picture:



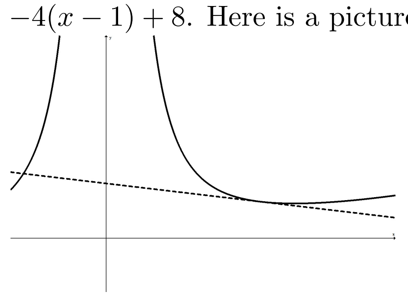
2. Find the equation for the tangent line to $y = 7e^x + 3$ at $x = 0$.

   *SOLUTION:* At $x = 0$, we have $y(0) = 7e^0 + 3 = 10$. Thus, the line goes through the point $(0, 10)$ and is of the form $y = m(x - 0) + 10$.
   Next, $y' = 7e^x$. Thus, the slope at $x = 0$ is $y'(0) = 7e^0 = 7$.
   Therefore, the tangent line is $y = 7(x - 0) + 10$. Here is a picture:



3. Find the equation for the tangent line to $y = \frac{4}{x^2} + (x + 1)^2$ at $x = 1$.

   *SOLUTION:* At $x = 1$, we have $y(1) = \frac{4}{(1)^2} + (1 + 1)^2 = 8$. Thus, the line goes through the point $(1, 8)$ and is of the form $y = m(x - 1) + 8$.
   Next, $y = 4x^{-2} + x^2 + 2x + 1$, so $y' = -8x^{-3} + 2x + 2$. Thus, the slope at $x = 1$ is $y'(1) = -8 + 2 + 2 = -4$.
   Therefore, the tangent line is $y = -4(x - 1) + 8$. Here is a picture:



4. Find the equations of the two lines that are tangent to $y = x^2$ and also pass through (0,-6).

   *SOLUTION:* First label the unknown tangent points by $(a, b)$. Now we write down all the conditions we are trying to satisfy:

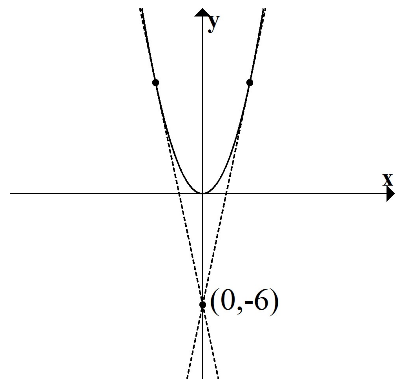
   (a) $(a, b)$ is on the curve. Thus, $b = a^2$.
   (b) The slope of the tangent is always $y' = 2x$, so at $(a, b)$ the SLOPE OF TANGENT $= 2a$.
   (c) The desired line needs to go through $(a, b)$ and $(0, -4)$, so DESIRED SLOPE $= \frac{b-(-6)}{a-0}$.

   We want the slope of the tangent at $(a, b)$ to match the desired slope, so we want to solve

$$2a = \frac{b + 6}{a} \qquad \text{and} \qquad b = a^2.$$

Now we simplify, combine and solve as follows: $2a^2 = b+6 \Rightarrow 2a^2 = a^2+6 \Rightarrow a^2 = 6 \Rightarrow a = \pm\sqrt{6}$.

The corresponding slopes are $-2\sqrt{6}$ and $2\sqrt{6}$. Therefore the two tangent lines are $y = -2\sqrt{6}(x-0)+(-6)$ and $y = 2\sqrt{6}(x-0)+(-6)$. Here is a picture:




5. Find the equation of the line that is tangent to $y = x^3$ and also pass through (0, 10).

   *SOLUTION:* First label the unknown tangent points by $(a, b)$. Now we write down all the conditions we are trying to satisfy:

   (a) $(a, b)$ is on the curve. Thus, $b = a^3$.

   (b) The slope of the tangent is always $y' = 3x^2$, so at $(a, b)$ the SLOPE OF TANGENT $= 3a^2$.

   (c) The desired line needs to go through $(a, b)$ and $(0, 10)$, so DESIRED SLOPE $= \frac{b-(10)}{a-0}$.

   We want the slope of the tangent at $(a, b)$ to match the desired slope, so we want to solve

$$3a^2 = \frac{b+10}{a} \qquad \text{and} \qquad b = a^3.$$

   Now we simplify, combine and solve as follows: $3a^3 = b+10 \Rightarrow 3a^3 = a^3+10 \Rightarrow 2a^3 = 10 \Rightarrow a = \pm\sqrt[3]{5}$.

   The corresponding slope is $y' = 3(5)^{2/3}$. Therefore the tangent line is $y = 3(5)^{2/3}(x-0)+10$. Here is a picture: