Math 120 Exam 2 One-Page Review: Chapters 8 through 16

This review highlights core skills. You are expected to know how to do all problems similar to those in the homework.

1. **Chapters 8 and 9: Composition and Inverses**
   - Compute compositions: $f(g(x))$, $g(f(x))$, $f(f(x))$.
   - A function has an inverse if it passes the horizontal line test.
   - To find an inverse: start with $y = f(x)$, solve for x in terms of y, then switch x and y.
   - Be able to solve for x in linear, rational, quadratic (may require restricted domain), and root-based expressions.

2. **Chapters 10, 11, 12: Exponential Models and Logarithms**
   - Model forms:
   $$y = y_0 b^x \qquad \text{and} \qquad y = y_0 e^{kx}, \text{ where } b = e^k.$$
   - Key log rules:
   $$\ln(a^r) = r\ln(a), \quad \ln(ab) = \ln(a) + \ln(b), \quad \ln(a/b) = \ln(a) - \ln(b).$$
   - Use logarithms to solve equations where the variable is in the exponent.
   - Doubling/tripling time depends on the growth rate, not the initial amount.
   - Change of base:
   $$\log_b(x) = \frac{\ln(x)}{\ln(b)}.$$

3. **Chapter 13: Moving Functions Around**
   - Reflect across y-axis: replace x by $-x$.
   - Reflect across x-axis: replace y by $-y$.
   - Shift horizontally by h: replace x by $x - h$.
   - Shift vertically by k: replace y by $y - k$.
   - Dilate horizontally by c: replace x by cx.
   - Dilate vertically by d: replace y by dy.
   - Best method: Mark points on the original graph, transform the coordinates, then re-plot.

4. **Chapter 14: Linear-to-Linear Models**
   - Standard form:
   $$y = \frac{ax + b}{x + c}.$$
   - Vertical asymptote: $x = -c$.
   - Horizontal asymptote: $y = a$.
   - After drawing asymptotes, the basic shape resembles $y = \pm 1/x$.

5. **Chapter 15: Arc Length and Wedge Area**
   - 2π radians $= 360°$.
   - If θ is in *radians*:
   $$s = r\theta, \qquad \text{Area} = \frac{1}{2}r^2\theta.$$
   - If θ is in *degrees*:
   $$s = \frac{\pi}{180}r\theta, \qquad \text{Area} = \frac{\pi}{360}r^2\theta.$$

6. **Chapter 16: Circular Motion and Belts**
   - Angular speed: $\omega = \frac{\theta}{t}$.
   - Linear speed: $v = \frac{s}{t}$.
   - When θ is in radians:
   $$s = r\theta, \qquad \theta = \omega t, \qquad v = \omega r.$$
   - In belt systems, all contact points on the belt have the same linear speed (but wheels may have different angular speeds depending on radius).