Math 120 Exam 2 Review: Chapters 8 through 16

This review is not all inclusive. You are expected to understand concepts from lecture and be able to complete problems similar to those from the homework.

1. **Chapters 8 and 9 — Composition and Inverses**

   - Be able to compute compositions such as $f(g(x))$, $g(f(x))$, and $f(f(x))$.
   - A function has an inverse only if it is one-to-one (passes the Horizontal Line Test).
   - To find an inverse:
     $$y = f(x) \quad \text{swap } x \text{ and } y \text{ and solve for } y.$$
   - The domain of f^{-1} is the range of f, and the range of f^{-1} is the domain of f.
   - A function and its inverse are reflections of each other across the line $y = x$, when drawn on the same coordinate plane.
   - Be comfortable solving for x when equations involve:
     (a) Linear expressions
     (b) Rational expressions
     (c) Quadratics
     (d) Square roots or other fractional powers
   - Example of restricting domain:
     Consider $f(x) = (x-2)^2 + 1$. This function is not invertible on all real x because it fails the Horizontal Line Test. If we restrict to $x \leq 2$, then:
     $$y = (x-2)^2 + 1 \ \Rightarrow\ (x-2)^2 = y - 1 \ \Rightarrow\ x - 2 = \pm\sqrt{y-1} \ \Rightarrow\ x = 2 \pm \sqrt{y-1}.$$
     Because we are restricting to $x \leq 2$, we choose the branch that gives x-values less than 2:
     $$f^{-1}(y) = 2 - \sqrt{y-1}, \quad \text{Domain: } y \geq 1.$$

2. **Chapters 10–12 — Exponential Models and Logarithms**

   - Exponential model:
     $$y = ab^t \qquad \text{or equivalently} \qquad y = ae^{kt},$$
     where $b = e^k$ and $k = \ln(b)$.
   - You should be very comfortable finding an exponential model from given information. Write out the model, plug in the numbers, then combine and solve.
   - Understand how to use doubling, tripling, or halving times to find b.
   - Note that growth of 3.2% per year means $b = 1.032$. Example:
     $$y = 100(1.032)^t.$$
   - To solve exponential equations:
     (a) Isolate the exponential.
     (b) Take natural logs of both sides.
     (c) Use $\ln(a^b) = b\ln(a)$ to solve for the variable.
   - A few other key facts:
     - $\ln(a^b) = b\ln(a)$
     - $\ln(ab) = \ln(a) + \ln(b)$
     - $\log_b(y) = x$ is the same as $y = b^x$
     - $\log_b(x) = \dfrac{\ln(x)}{\ln(b)}$

3. **Chapter 13 — Transforming Functions**

- Movements:
  (a) Reflect across y-axis: replace x with $-x$.
  (b) Reflect across x-axis: replace y with $-y$.
  (c) Shift horizontally by h: replace x with $x - h$.
  (d) Shift vertically by k: replace y with $y - k$.
  (e) Horizontal scale by c: replace x with cx.
  (f) Vertical scale by d: replace y with dy.
- Example: $y = 2f(3x - 1) + 4$ Move outside stuff to y side: $\frac{y-4}{2} = f(3x - 1)$. To find the new x-coordinates: add 1 then divide by 3 (we are *undoing* the inside changes). To find new y-coordinates: multiply by 2, then add 4.
- Example of point movement: If $(2, 3)$ is on the original $y = f(x)$, then:

$$x: \ 2 \mapsto \frac{2+1}{3} = 1, \qquad y: \ 3 \mapsto 2(3) + 4 = 10.$$

  So $(2, 3)$ becomes $(1, 10)$ on the new graph.

4. **Chapter 14 — Linear-to-Linear Rational Models**

- Standard form:

$$y = \frac{ax + b}{x + c}.$$

- Vertical asymptote: $x = -c$.
- Horizontal asymptote: $y = a$ (ratio of leading coefficients).
- Once asymptotes are drawn, the graph resembles either $1/x$ or $-1/x$.
- Be comfortable solving for a, b, and c given data.
- When solving equations, clear the denominator first.

5. **Chapter 15 — Radians, Arc Length, and Wedge Area**

- 2π radians $= 360°$.
- If θ is in radians:

$$\text{Arc Length } = \theta r, \qquad \text{Wedge Area } = \frac{1}{2}\theta r^2.$$

- Always check whether θ is in radians before using formulas.

6. **Chapter 16 — Circular Motion and Belt/Wheel Systems**

- Angular speed: $\omega = \frac{\theta}{t}$.
- Linear speed: $v = \frac{s}{t}$.
- Key relationships (only if θ is in radians):

$$s = \theta r, \qquad \theta = \omega t, \qquad v = \omega r.$$

- Convert revolutions to radians: 1 rev $= 2\pi$ radians.
- Distance per revolution (circumference) $= 2\pi r$.
- In belt and wheel systems,
  - same axle means same angular speed and
  - connected by a belt means same linear speed.
- Use your table method once units are consistent.