

Math 120 – Ch 6 & 7 Test Prep – Dr. Loveless

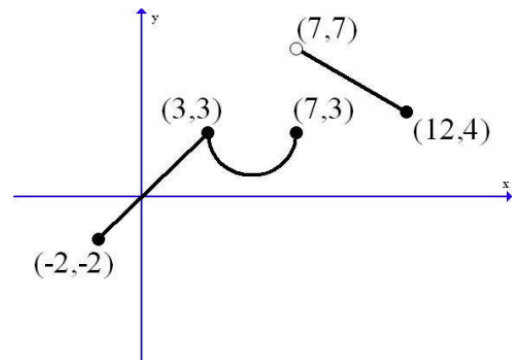
Take 5 minutes to try to complete this problem. Grade yourself to see how you did. Then discuss the solution with your TA. Then get to work on homework... ask lots and lots of chapter 7 questions!

Participation Code: Please remember to ask your TA for the participation code for today and don't forget to enter it in the corresponding Canvas quiz.

[F06 Exam 1 - Loveless]

Problem 1: Let $y = f(x)$ be the function given by the graph below which consists of two line segments and a lower semicircle. **(see the bottom of this page for a written description of the graph).*

a) Find the multipart rule for $f(x)$.



b) Compute $f(4)$.

c) *Challenge (this wasn't on the test, but I think you can do it!)*
Find all solutions to $f(x) = 2$.

**written description of graph*

Graph has three parts:

- a line segment from $(-2, -2)$ to $(3, 3)$ with filled endpoints;
- a lower semicircle from $(3, 3)$ to $(7, 3)$ centered at $(5, 3)$ with filled endpoints;
- a line segment from $(7, 7)$ to $(12, 4)$ with an open endpoint at $(7, 7)$ and filled endpoint at $(12, 4)$.

(10 pts) Problem 1:

a) Find the multipart rule for $f(x)$.

From $x = -2$ to $x = 3$, the function is a line through $(-2, -2)$ and $(3, 3)$ so $m = \frac{3 - (-2)}{3 - (-2)} = 1$, and the equation of the line is $y = (x - (-2)) - 2 = x$. (+1 line equation)

From $x = 3$ to $x = 7$, the points are on a circle centered at $(5, 3)$ of radius 2. The equation satisfied by all points on the circle is $(x - 5)^2 + (y - 3)^2 = 4$. Solving for y and using the negative root gives the lower semicircle equation $y =$ (+2 semicircle equation)

From $x = 7$ to $x = 12$ (note that the line starts **strictly** after 7), the points are on a line through $(7, 7)$ and $(12, 4)$ so $m = \frac{4 - 7}{12 - 7} = -\frac{3}{5}$, and the equation of the line is $y = -\frac{3}{5}(x - 7) + 7$. (+1 line equation)

$$f(x) = \begin{cases} x & , \text{ if } -2 \leq x < 3 \\ 3 - \sqrt{4 - (x - 5)^2} & , \text{ if } 3 \leq x < 7 \\ \text{ } & , \text{ if } 7 < x \leq 12. \end{cases}$$
(+2 correct notation)

b) Compute $f(8)$.

Since $x = 8$ is between 7 and 12, we use the last rule, so we get (+1 correct evaluation)

$$f(8) = -\frac{3}{5}(8 - 7) + 7 =$$

c) Find all solutions to $f(x) = 2$.

We can see from the picture that the function will equal 2 once between $x = -2$ and $x = 3$ and twice between $x = 3$ and $x = 7$. So we need to solve...

When $y = 2$ in $y = x$, so $x = 2$. (+1 found $x = 2$)

When $y = 2$ in $y = 3 - \sqrt{4 - (x - 5)^2}$, we get $3 - \sqrt{4 - (x - 5)^2} = 2$, which means

- $\sqrt{4 - (x - 5)^2} = 1$, which gives (+1 found set up equation)
- $4 - (x - 5)^2 = 1$, which gives

(+1 correctly solved)
