## Chapters 6: Graphs and Multipart Functions

This document provides a quick summary and review for Chapters 6.

### Key Concepts

- Understand and describe when a graph is positive, negative, increasing, or decreasing.
- Label graphs using function notation and interpret what $f(x)$ represents at specific points.
- Work with piecewise-defined functions, such as

$$|x| = \begin{cases} x, & x \geq 0, \\ -x, & x < 0. \end{cases}$$

- Apply piecewise reasoning to geometric or modeling problems.

### Mechanical Examples of Multipart Functions

**Example 1:** Solve for a specific function value
Let

$$f(x) = \begin{cases} x^2 + 2, & x \geq 2, \\ 2x - 1, & x < 2. \end{cases}$$

Find all $x$ such that $f(x) = 11$.

***Solution:***

- *Case 1:* $x \geq 2$ $\quad x^2 + 2 = 11 \implies x^2 = 9 \implies x = 3$ (valid since $x \geq 2$).
- *Case 2:* $x < 2$ $\quad 2x - 1 = 11 \implies x = 6$ (invalid since $x \not< 2$).

**Answer:** $x = 3$.

---

**Example 2:** Solve with an absolute value function
Let

$$g(x) = 4x - 5 + |x + 7|.$$

Find all $a$ such that $g(a) = 3a + 6$.

***Solution:***

- *Case 1:* $x + 7 \geq 0$ $(x \geq -7)$ $\quad |x + 7| = x + 7$, so $g(x) = 4x - 5 + (x + 7) = 5x + 2$.
  $5a + 2 = 3a + 6 \implies 2a = 4 \implies a = 2$ (valid).
- *Case 2:* $x + 7 < 0$ $(x < -7)$ $\quad |x + 7| = -x - 7$, so $g(x) = 4x - 5 - x - 7 = 3x - 12$.
  $3a - 12 = 3a + 6$ gives $-12 = 6$ so no solution.

**Answer:** $a = 2$.

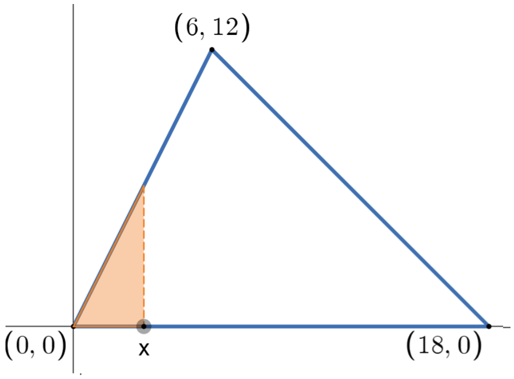
---

**Observation:**

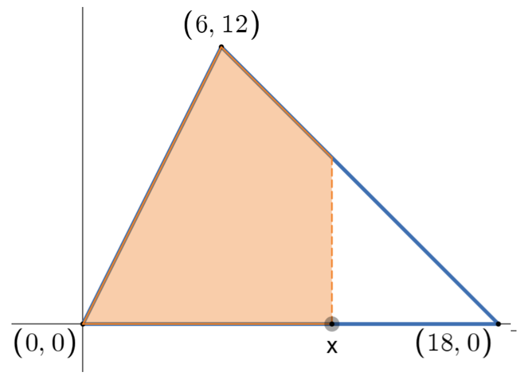
- Always split work by the region of definition.
- Check each answer against its case's domain.

**Example: Cake Cutting**

A triangular cake viewed from above has base 18 in and height 12 in. Coordinates: left corner $(0, 0)$, right corner $(18, 0)$, apex $(6, 12)$. Alice cuts vertically at $x$, taking the left portion.



Case 1: $0 \leq x \leq 6$ — Left side of the triangle



Case 2: $6 \leq x \leq 18$ — Right side of the triangle

***Solution:***

**Step 1: Equation for top edge.**

- Left edge from $(0, 0)$ to $(6, 12)$: slope $m = \frac{12-0}{6-0} = 2$, so the equation is $y = 2x$.
- Right edge from $(6, 12)$ to $(18, 0)$: slope $m = \frac{0-12}{18-6} = -1$, so the equation is $y = -x + 18$.

**Step 2: Height of the slice at cut $x$.** As Alice moves the knife from left to right, the top height changes depending on which edge she intersects:

$$y(x) = \begin{cases} 2x, & 0 \leq x \leq 6, \\ -x + 18, & 6 \leq x \leq 18. \end{cases}$$

**Step 3: Area of shaded portion.**

- *Case 1:* For $0 \leq x \leq 6$, the region is a triangle and area is $\frac{1}{2}$(base)(height), so

$$A(x) = \tfrac{1}{2}x(2x) = x^2.$$

- *Case 2:* For $6 \leq x \leq 18$, find the total area of the cake and subtract the small right triangle's area:

$$A(x) = \tfrac{1}{2}(18)(12) - \tfrac{1}{2}(18 - x)(-x + 18) = 108 - \tfrac{1}{2}(18 - x)^2.$$

Thus,

$$A(x) = \begin{cases} x^2, & 0 \leq x \leq 6, \\ 108 - \frac{1}{2}(18 - x)^2, & 6 \leq x \leq 18. \end{cases}$$

**Observations:**

- Break geometry problems into straight-line equations first.
- Clearly define variables: here $x$ = base coordinate, $y(x)$ = height at the cut, $A(x)$ = area to the left.