Chapter 3 Summary: Circles and Horizontal/Vertical Lines

This document provides a quick summary and review for Chapter 3.

Key Facts

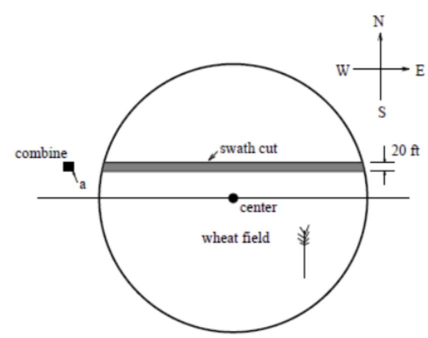
- Circle Equation: $(x-h)^2+(y-k)^2=r^2$
  - (h,k) is the center and r is the radius.
- Rearranging to solve for x or y gives the semicircles:
  - Upper semicircle: $y=k+\sqrt{r^2-(x-h)^2}$; Lower semicircle: $y=k-\sqrt{r^2-(x-h)^2}$.
  - Right semicircle: $x=h+\sqrt{r^2-(y-k)^2}$; Left semicircle: $x=h-\sqrt{r^2-(y-k)^2}$.
- Horizontal line: All points (x,y) where $y=a$ gives a horizontal line at $y=a$.
- Vertical line: All points (x,y) where $x=b$ gives a vertical line at $x=b$.

Tips

- **Be able to find the intersection of a line and circle**.
  - This involves "combining conditions" (substitute to get one variable).
  - Don't forget the ideas from chapter 1 and 2, in particular time $=\frac{\text{dist}}{\text{speed}}$.

Example: Nora harvest wheat (Problem 3.7)

Nora spends part of her summer driving a *combine* during wheat harvest. Assume she starts at the indicated position heading east at 10 ft/sec toward a circular wheat field of radius 300 ft. The combine cuts a swath 20 feet wide and begins when the corner of the machine labeled "a" is 90 feet north and 90 feet west of the western-most edge of the field.



Question:

(a) When does Nora's rig first start cutting the wheat?

Solutions (let's use our V.E.T.S. method)

- Visualize - Draw a coordinate system and label. I would set the origin at the center of the circle.
- Equations
  - Points on the edge of the circular field: $x^2+y^2=300^2$
  - Lower part of swath: $y=90$.
- Translates - Find how long it takes to get to the edge:
  - Recall time $=\frac{\text{dist}}{\text{speed}}$.
  - Speed is 10 ft/sec, so time $=\frac{\text{dist}}{10}$.
- Solve - Substitute $y=90$ into the circle:

$$x^2+90^2=300^2 \quad\Longrightarrow\quad x^2=90000-8100=81900$$

$$x=\pm\sqrt{81900}\approx\pm 286.182$$

  Corner 'a' started at $x=-390$ and reaches $x=-286.182$. Time to get there:

$$t\approx\frac{390-286.18}{10}\approx 10.38 \text{ seconds.}$$