Chapter 16: Circular Motion

This document provides a quick summary and review for Chapter 16.

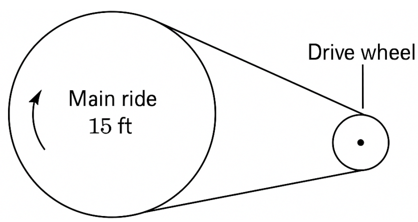
Key Concepts

Key Facts / What You Need to Know

- **Angular speed:** $\omega = \dfrac{\text{angle change}}{\text{time change}}$.
- **Linear speed:** $v = \dfrac{\text{distance change}}{\text{time change}}$.
- **Use radians in all formulas:**
$$s = \theta r, \qquad \theta = \omega t, \qquad v = \omega r.$$
- **Conversions:** $1 \text{ rev} = 2\pi \text{ rad}$, $\omega_{\text{rad/min}} = 2\pi \cdot (\text{RPM})$.
- **Belt & Wheel Strategy:**
  - $v = \omega r$ connects linear and angular speeds (and is important, but you MUST be in radians).
  - Wheels on the **same axle** share the same ω.
  - Wheels connected by a **belt** share the same v.

Example 1: Ride + Drive Wheel

A spinning carnival ride is shown below. The main ride has radius 15 ft and is driven by a belt connected to a drive wheel that has radius 2.5 ft. Assume the main ride rotates at 48 RPM.



(a) Find the linear speed if you are on the outer edge of the ride (in ft/s and mph).

Solution
$\omega_{\text{ride}} = 48 \text{ RPM} = 96\pi \text{ rad/min}$.
$v_{\text{edge}} = \omega r = (96\pi) \cdot 15 = 1440\pi \text{ ft/min}$.
$\text{mph} = \dfrac{1440\pi \cdot 60}{5280} = \dfrac{180\pi}{11} \approx 51.4 \text{ mph}$.

(b) Find the angular speed of the drive wheel in RPM.

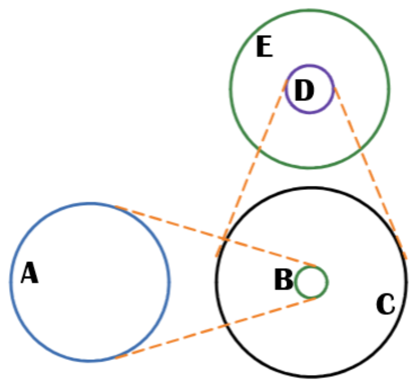
Solution
Belt $\Rightarrow$ same linear speed: $v_{\text{drive}} = 1440\pi \text{ ft/min}$.
$\omega_{\text{drive}} = \dfrac{v}{r} = \dfrac{1440\pi}{2.5} = 576\pi \text{ rad/min} = 288 \text{ RPM}$.

(c) Find the linear speed of a rider that is 10 ft from the center (in ft/s and mph).

Solution
$v = \omega r = (96\pi)10 = 960\pi \text{ ft/min} \approx 34.3 \text{ mph}$.

Example 2: Five–Wheel Belt System

Consider the belt and wheel system shown below. Belts connect A to B and C to D. Axles connect B to C and D to E. All radii are given as $r_A = 5$ in, $r_B = 1.5$ in, $r_C = 9$ in, $r_D = 2$ in, $r_E = 6$ in.



If the angular speed of wheel A is 24 RPM, find the linear speed of wheel E in in/min.

Blank table (students fill in):

Wheel	**Linear speed v (in/min)**	**Angular speed ω (rad/min)**	**Radius r (in)**
A			5
B			1.5
C			9
D			2
E			6

Solution on the next page. Once all angles are in radians and units match, this becomes a quick multiply/divide "fill-in-the-table" problem.

Example 2: Filled Table & Work (for reference)

Step chain (use $v = \omega r$):

$$\omega_A = 24 \text{ RPM} = 48\pi \text{ rad/min}, \quad v_A = r_A\omega_A = 5(48\pi) = 240\pi \text{ in/min}.$$
$$\text{Belt } A \leftrightarrow B: \ v_B = v_A = 240\pi \text{ in/min} \Rightarrow \omega_B = \frac{v_B}{r_B} = \frac{240\pi}{1.5} = 160\pi \text{ rad/min}.$$
$$\text{Same axle } (B \equiv C): \ \omega_C = \omega_B, \quad v_C = r_C(2\pi \cdot 80) = 9(160\pi) = 1440\pi \text{ in/min}.$$
$$\text{Belt } C \leftrightarrow D: \ v_D = v_C = 1440\pi \text{ in/min} \Rightarrow \omega_D = \frac{v_D}{r_D} = \frac{1440\pi}{2} = 720\pi \text{ rad/min}.$$
$$\text{Same axle } (D \equiv E): \ \omega_E = \omega_D, \quad v_E = r_E(2\pi \cdot 360) = 6(720\pi) = 4320\pi \text{ in/min}.$$

Filled table:

Wheel	**Linear speed v (in/min)**	**Angular speed ω (rad/min)**	**Radius r (in)**
A	240π	48π	5
B	240π	160π	1.5
C	1440π	160π	9
D	1440π	720π	2
E	4320π	720π	6

Linear speed of $E = 4320\pi$ in/min.

Observation: Notice that $v = \omega r$ for each wheel—the table makes this clear at a glance.