Chapter 15: Measuring an Angle

This document provides a quick summary and review for Chapter 15.

Key Concepts

- Conversions: 1 revolution $= 360° = 2\pi$ radians. Hence $\theta_{\text{rad}} = \left(\frac{\pi}{180}\right)\theta_{\text{deg}}$ and $\theta_{\text{deg}} = \left(\frac{180}{\pi}\right)\theta_{\text{rad}}$.
- **Arc length** on a circle of radius r:

$$\text{(radians)} \quad s = \theta r \qquad\qquad \text{(degrees)} \quad s = \left(\frac{\pi}{180}\right)\theta r.$$

- **Area of a circular sector (wedge)** of radius r:

$$\text{(radians)} \quad A = \tfrac{1}{2}\theta r^2, \qquad\qquad \text{(degrees)} \quad A = \left(\frac{\pi}{360}\right)\theta r^2.$$

Quick Example: Water-Treatment Arm

A water-treatment facility drips water from a 40 ft arm mounted at a central pivot. The arm completes one full revolution every 8 minutes.

(a) Find the area of charcoal covered after 1 minute.

Solution The fraction of one full revolution is $\frac{1}{8}$ of the circle. So the area will be one-eighth of the area of the entire circle.

$$A = \tfrac{1}{8}\pi r^2 = \tfrac{1}{8}\pi(40)^2 = 200\pi \text{ ft}^2 \approx 628.3 \text{ ft}^2.$$

Note that the fraction of the circle will be $\frac{t}{8}$ where t is in minutes. We will talk more about circular motion and speed in the next chapter. Also note, you could convert $\frac{1}{8}$ revolutions to degrees or radians and use the formulas above.

For example:

$$\frac{1}{8} \text{ rev} = \frac{2\pi}{8} \text{ rad} = \frac{\pi}{4} \text{ rad}$$

And using the formula you get...

$$A = \tfrac{1}{2}\,\frac{\pi}{4} r^2 = \tfrac{\pi}{8}(40)^2 = \frac{1600}{8}\pi = 200\pi \text{ ft}^2 \approx 628.3 \text{ ft}^2.$$

(b) Find the area covered after 1 second.

Solution Convert to minutes: 1 second $= \frac{1}{60}$ minute, so the fraction of a circle is $\frac{1/60}{8} = \frac{1}{480}$.

$$A = \tfrac{1}{480}\pi r^2 = \tfrac{1}{480}\pi(40)^2 = \frac{1600}{480}\pi = \frac{10}{3}\pi \text{ ft}^2 \approx 10.47 \text{ ft}^2.$$

(c) How long (in time) to cover 150 ft^2?

Solution Let's use radians. The formula for area covered is $A = \frac{1}{2}\theta r^2$, so $150 = \frac{1}{2}\theta(40)^2$ giving $\theta = \frac{150}{800} = \frac{3}{16}$ radians. We want to convert that to revolutions, then to time:

$$\frac{3}{16}\text{rad} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} \cdot \frac{8 \text{ min}}{1 \text{ rev}} = \frac{12}{16\pi} = \frac{3}{4\pi} \text{ minutes} \approx 0.2387 \text{ min} \approx 14.3 \text{ s}.$$