Chapter 13: Shifting, Reflecting, and Dilating Functions

This is a review about shifting, dilating, and reflecting functions. Students should also read the text for another, equivalent approach.

Key Facts

Formula	Effect	Formula	Effect
$y = f(x) + k$	Up by k	$y = f(x - h)$	Right by h
$y = -f(x)$	Flip vertically (x−axis)	$y = f(-x)$	Flip horizontally (y−axis)
$y = a\,f(x)$	Multiply y by a (vertical scale)	$y = f(bx)$	Multiply x by $1/b$ (horizontal scale)

How to handle multiple operations

We're answering: given a base graph $y = f(x)$, how do we quickly graph something like

$$y = 2f(3x - 4) - 5\,?$$

Below are two fully equivalent methods. Pick whichever "clicks" for you.

Method 1: y-stuff to the y-side
Rewrite so the y-stuff is isolated:

$$\frac{1}{2}(y + 5) = f(3x - 4).$$

Undo the changes...

- $\dfrac{(3x - 4 + 4)}{3} = x.$
- $\dfrac{1}{2}(y + 5) \cdot (2) - 5 = y.$

Method 2: 'new' in terms of 'old'
Rewrite

$$y_{\text{old}} = f(x_{\text{old}}), \qquad y_{\text{new}} = 2f(3x_{\text{new}} - 4) - 5.$$

Match the insides and solve:

$$3x_{\text{new}} - 4 = x_{\text{old}} \quad \Longrightarrow \quad x_{\text{new}} = \frac{x_{\text{old}} + 4}{3}.$$

Then note

$$y_{\text{new}} = 2f(x_{\text{old}}) - 5 = 2y_{\text{old}} - 5.$$

In both methods we reach the same result:
Horizontal changes (from $3x - 4$):

1. Add 4 to every x-coordinate. (right by 4)
2. Divide by 3 (horizontal shrink by factor 3).

Vertical changes (from $\frac{1}{2}(y + 5)$):

1. Multiply y by 2 (vertical stretch by 2).
2. Subtract 5 (shift down 5).

Apply those steps to each chosen point from the original graph. Plot the results.

Example: Apply both methods to a quadratic

We'll start with the base function

$$f(x) = x^2,$$

whose graph includes the easy points

$$(0,0), \quad (2,4), \quad (-2,4).$$

Now consider the transformed function

$$g(x) = -2\,f(x-4) + 5 = -2(x-4)^2 + 5.$$

We'll show how both methods describe the same movement.

Method 1: y-stuff to the y-side
Rewrite to isolate y:

$$\frac{1}{-2}(y-5) = f(x-4).$$

Undo the changes...

- $(x-4)+4 = x.$
- $\frac{1}{-2}(y-5)\cdot(-2)+5 = y.$

Method 2: 'new' in terms of 'old'
Rewrite

$$y_{\text{old}} = f(x_{\text{old}}), \qquad y_{\text{new}} = -2f(x_{\text{new}}-4)+5.$$

Thus,

$$x_{\text{new}} - 4 = x_{\text{old}} \quad \Longrightarrow \quad x_{\text{new}} = x_{\text{old}} + 4,$$

$$y_{\text{new}} = -2f(x_{\text{old}}) + 5 = -2y_{\text{old}} + 5.$$

Points:

1. $(0,0)$ becomes $(4,5)$
2. $(2,4)$ becomes $(6,-3)$
3. $(-2,4)$ becomes $(2,-3)$

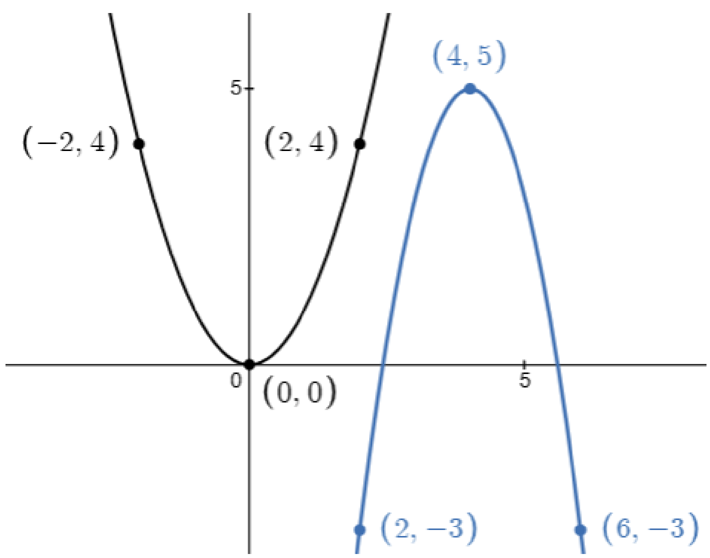

Figure 1: Old $y = x^2$ (black) and new $y = -2(x-4)^2 + 5$ (blue).