Chapter 12: Logarithmic Functions

This document provides a concise review of logarithmic functions.

Key Concepts

- Logs are the inverses of exponentials:

$$y = e^x \iff x = \ln(y), \qquad y = b^x \iff x = \log_b(y) = \frac{\ln(y)}{\ln(b)}.$$

- Logarithm properties:

$$\ln(ab) = \ln a + \ln b, \quad \ln\left(\frac{a}{b}\right) = \ln a - \ln b, \quad \ln(a^t) = t\ln a.$$

- Change of base: $\log_b(x) = \dfrac{\ln(x)}{\ln(b)}$.
- Exponential functions can be rewritten using e: $A_0 b^t = A_0 e^{(\ln b)t}$.

Example 1: Logarithmic Simplification

Simplify each expression of the expressions $\ln(8^3)$, $\ln\sqrt{x}$, and $\ln\left(\frac{3}{5}\right)$.

Solution:

$$\ln(8^3) = 3\ln(8), \qquad \ln\sqrt{x} = \frac{1}{2}\ln(x), \qquad \ln\left(\frac{3}{5}\right) = \ln(3) - \ln(5).$$

Example 2: Solving an Exponential Equation

Solve $10 + 3^x = 90$.

Solution:

$$3^x = 80 \Rightarrow \ln(3^x) = \ln(80) \Rightarrow x\ln(3) = \ln(80) \Rightarrow x = \frac{\ln(80)}{\ln(3)} \approx 3.989.$$

Example 3: Continuous Compounding

Solve for t: $130{,}000 = 2{,}000e^{0.064t}$.

Solution:

$$65 = e^{0.064t} \Rightarrow \ln(65) = 0.064t \Rightarrow t = \frac{\ln(65)}{0.064} \approx 65.22 \text{ years.}$$

Example 4: Rewriting Exponential Models

Rewrite $200(2^t)$ using e.

Solution:

$$2^t = e^{t\ln(2)} = e^{0.69315t} \Rightarrow 200(2^t) = 200e^{0.69315t}.$$