Ch 9: Inverse Functions Video Link

Goal: Learn the general ideas of inverse functions

Entry Task (motivation) A ball is thrown in the air. The height in feet after x seconds is given by $y = f(x) = -16x^2 + 43x$.

Try to answer the following questions…

1. "Plug in question" What is the height after 1 second?
2. "Inverse question" When is the ball 20 feet high?
3. *Challenge*: Can you solve for x in terms of y in general?

General Notes and Observations

Q: The sol'n for x in terms of y is NOT a function in the last problem, why?

In general: A parabola does not have one inverse ***function***.

Horizontal Line Test and One-to-One

We say that parabolas function is not ***one-to-one***. Meaning there is not just one x-value for every y-value. Visually, a one variable function is one-to-one if it passes the **horizontal line test**.

You do:

- Draw an example of a curve that is not a function.

- Draw two examples of functions that are one-to-one.

- Draw two examples of functions that are not one-to-one.

Notation:

When a function $y = f(x)$ is one-to-one, then we say it is ***invertible*.** That means we can solve for x in terms of y and get one function. We call this function the inverse and write

$$x = f^{-1}(y).$$

Example 1: Find the inverse of

$$y = f(x) = 5x + 1.$$

Checking work:

Plug in x = 2 to $y = f(x)$, then plug that y into $x = f^{-1}(y)$.

What happens?

Observation: If are answer is correct, then

$$f^{-1}(f(x)) = x \text{ and } f(f^{-1}(y)) = y$$

Example 2: Find the inverse of

$$y = g(x) = \frac{5}{x + 3}.$$

Intermission – A brief review of inverse pairs

Do you know your current toolbox of solving skills?

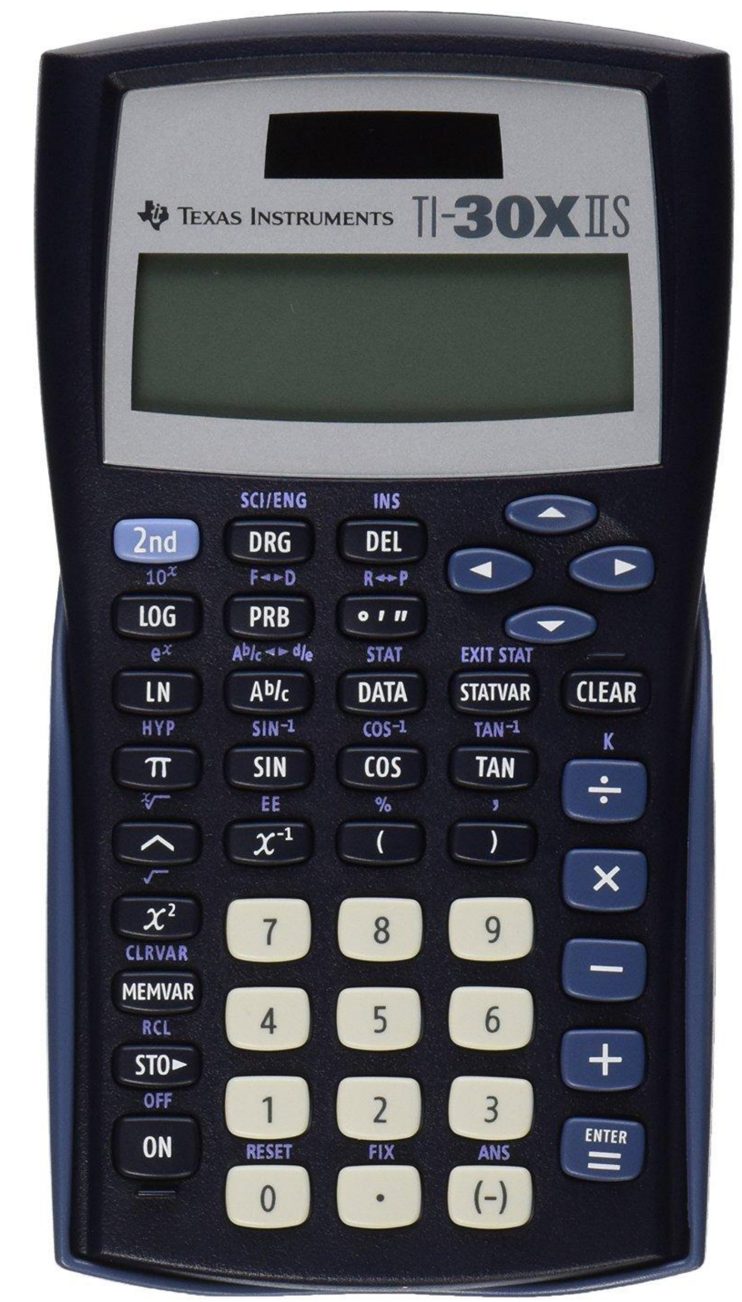
Fill in the table below…

Equation	Inverse?
$x + 3 = 14$	$x =$
$y - 5 = 22$	$y =$
$3t = 16$	$t =$
$\frac{m}{0.2} = 100$	$m =$
$x^2 = 7$	$x =$
$\sqrt{y} = 3$	$y =$
$t^5 = 20$	$t =$
$\sqrt[5]{w} = 3$	$w =$

Example: If you know your inverses you can solve this by pushing five operations buttons in the correct order on your calculator.

Try it…

$$\sqrt[3]{(2x-1)^5 - 5} = 3$$



Non-invertible functions and how we split them up

If a function is not one-to-one, we typical split the domain into parts where the function is one-to-one and give an inverse for each part.

For each example below: Split up into two intervals, find both inverses, give domains.

Graphical note: The inverse function is the mirror image of the function reflected across the line $y = x$.

Domain note: The domain of the inverse is the same as the range of the original function

Example 1: $y = g(x) = x^2$

Example 2: $y = f(x) = -16x^2 + 43x$

Example 3: $y = h(x) = \sqrt{4 - x^2}$

Homework Challenges…

(c) $g(x) = 3\sqrt{5 - x} - 6$

$g^{-1}(x) =$

Domain:

- ○ $0 \le x < \infty$
- ○ $-\infty < x \le 0$
- ○ $-6 \le x < \infty$
- ○ $-\infty < x < \infty$
- ○ $0 \le x \le 4$

(d) $j(x) = \sqrt{x} + \sqrt{x - 1}$

$j^{-1}(x) =$

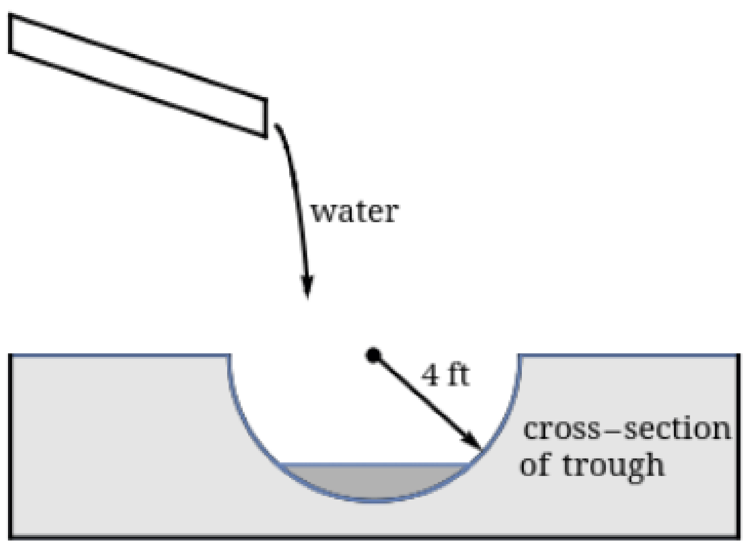
Domain:

- ○ $1 \le x < \infty$
- ○ $-\infty < x < \infty$
- ○ $0 \le x < \infty$
- ○ $-6 \le x < \infty$
- ○ $0 \le x \le 4$

Homework Challenges

5. [– / 14 Points]

A trough has a semicircular cross section with a radius of 4 feet. Water starts flowing into the trough in such a way that the depth of the water is increasing at a rate of 3 inches per hour.



(a) Give a function $w = f(t)$ relating the width w, in feet of the surface of the water to the time t, in hours. Make sure to specify the domain and compute the range too.

$w =$

Domain:

- ○ $0 \le t \le 16$
- ○ $-8 \le t \le 8$
- ○ $-16 \le t \le 32$
- ○ $0 \le t \le 8$
- ○ $-\infty < t < \infty$