

Course Notes We've learned lines and quadratics as well as functional notation and graphs. We've used these tools in applied settings involving distance, speed, time, and circles.

What's up for the next two weeks: Ch 8-9: Compositions and inverses. Ch 10-12: Exponentials and logarithms.

- The composition of two functions is $(f \circ g)(x) = f(g(x))$.
- To compute $f(g(x))$, replace every occurrence of x in $f(x)$ with $g(x)$.
- Notes on domain and range:
 - The domain of $f(g(x))$ is contained in the domain of $g(x)$ as the input to g hasn't changed.
 - The range of $f(g(x))$ is contained in the range of $f(x)$ as the output is still from f .

Ch 8: Composition

Goal: Learn composition of functions.

Entry Task

Consider $f(x) = 2x + 5$, $g(x) = x^2 - x + 1$, and $h(x) = \sqrt{x}$.

- Compute $f(g(2))$
- Find a formula for $f(g(x))$.
- Find formula $h(f(x))$ and $g(h(x))$ and $f(f(x))$.

Can you do these Ch 8 homework problems now?

1. [- / 12 Points]

Write each of the following functions as a composition of two functions f and g .

(a) $y = (x - 11)^5$
 $(f(x), g(x)) = (\text{ })$

(b) $y = \sqrt[5]{1 + x^4}$
 $(f(x), g(x)) = (\text{ })$

(c) $y = 3(x - 3)^5 - 6(x - 3)^2 + \frac{1}{2}(x - 3) + 12$
 $(f(x), g(x)) = (\text{ })$

(d) $y = \frac{1}{x^2 + 2}$
 $(f(x), g(x)) = (\text{ })$

2. [- / 24 Points]

Compute the compositions $f(g(x))$, $f(f(x))$ and $g(f(x))$.

(a) $f(x) = x^2, g(x) = x + 5$
 $f(g(x)) = \text{ }$
 $f(f(x)) = \text{ }$
 $g(f(x)) = \text{ }$

(b) $f(x) = 1/x, g(x) = \sqrt{x}$
 $f(g(x)) = \text{ }$
 $f(f(x)) = \text{ }$
 $g(f(x)) = \text{ }$

(c) $f(x) = 4x + 9, g(x) = \frac{1}{4}(x - 9)$
 $f(g(x)) = \text{ }$

Example (like homework)

(a) If $f(x) = mx + b$, then find and simplify $f(f(x))$.

(b) *Small Challenge:* If I told you that $f(f(x)) = 9x + 5$, then can you deduce the value(s) of m and b

Domain and composition

Consider $f(x) = 2x + 5$ and $h(x) = \sqrt{x}$.

- (a) Find the domain of $h(x)$.
- (b) Find the domain of $h(f(x))$.
- (c) Find the domain of $f(h(x))$

Any Observations?

Can you do this homework problem now?

the domain of $f(x)$ is $1 \leq x \leq 5$ and the range of $f(x)$ is $-2 \leq y \leq 6$.

- (a) What is the domain of $f(2(x - 3))$?

☐ $0 \leq x \leq 4$

☐ $\frac{7}{2} \leq x \leq \frac{15}{2}$

☐ $\frac{7}{2} \leq x \leq \frac{11}{2}$

☐ $\frac{3}{2} \leq x \leq \frac{11}{2}$

☐ $1 \leq x \leq 5$
- (b) What is the range of $f(2(x - 3))$?

☐ $-2 \leq y \leq 8$

☐ $-4 \leq y \leq 6$

☐ $-7 \leq y \leq 1$

☐ $-2 \leq y \leq 6$

☐ $3 \leq y \leq 11$

- (c) What is the domain of $2f(x) - 3$?

☐ $-1 \leq x \leq 5$

☐ $-4 \leq x \leq 0$

☐ $6 \leq x \leq 10$

☐ $1 \leq x \leq 7$

☐ $1 \leq x \leq 5$
- (d) What is the range of $2f(x) - 3$?

☐ $-2 \leq y \leq 14$

☐ $-7 \leq y \leq 11$

☐ $-7 \leq y \leq 9$

☐ $-12 \leq y \leq 4$

☐ $-9 \leq y \leq 9$

Composition and Multipart Functions

Example: $f(x) = 3x - 12$ and $g(x) = |x| = \begin{cases} -x, & \text{if } x < 0; \\ x, & \text{if } x \geq 0. \end{cases}$

Find the multipart rule for

(a) $f(g(x))$

(b) $g(f(x))$

Example: $h(x) = x + 5$ and $p(x) = \begin{cases} x + 10, & \text{if } x < -1; \\ x^2, & \text{if } x \geq -1. \end{cases}$

Find the multipart rule for

(a) $p(h(x))$

(b) $h(p(x))$

Challenge Problem:

Find the multipart rule for

$$||x| - 3|$$