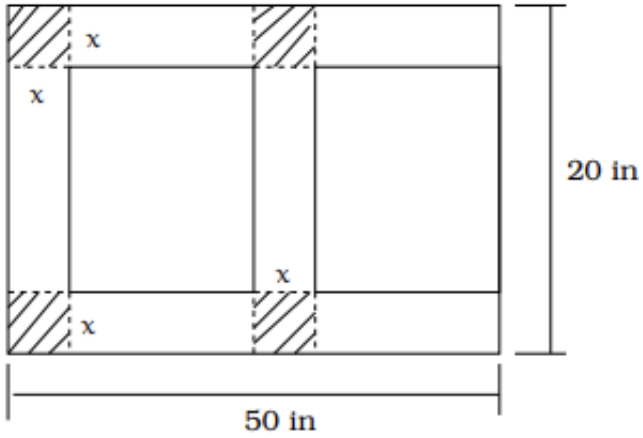
Ch 7: Quadratics

but first…

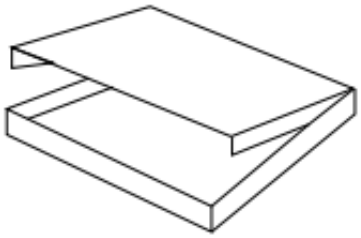
Entry Task on "*polynomials*"

A 20 inch by 50 inch piece of cardboard is going to be used to make a box.

The box will be made by cutting out four squares with side length x as shown.



remove shaded squares and fold to get:



For both parts below find the function in terms of x.

(a) Find the volume, V(x), of the resulting box

(b) Find the exposed surface area, A(x)?

Ch 7: Quadratics

Goal: Learn about second degree polynomials, which are great for modeling the path of a thrown ball and for applied max/min problems.

$y = ax^2 + bx + c$ (general form)

$y = a(x - h)^2 + k$ (vertex form)

Key Facts

- $h = -\frac{b}{2a}$ (x-coord. of vertex)
- $a > 0$ ⇒ open upward (smile)
- $a < 0$ ⇒ open downward (frown)
- $a = 0$ ⇒ not a quadratic!

Examples 1: Plot points and graph

$$y = x^2$$

***Examples 2*:** Plot points and graph

$$y = -2(x + 1)^2 + 2$$

Completing the Square and the Vertex Derivation

Expanding $y = a(x-h)^2 + k$ gives $y = ax^2 - 2ahx + ah^2 + k$

Example: Find $a, h,$ and k if

$$y = 3x^2 - 2x + 1$$

Example: Find $a, h,$ and k

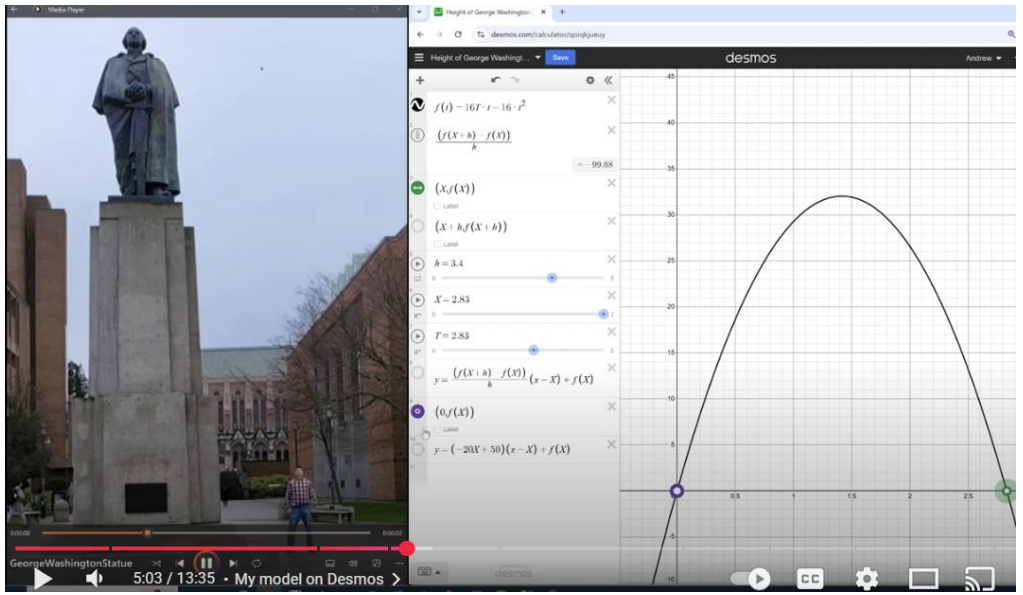
$$y = ax^2 + bx + c$$

Example *(height of Washington Statue)*:

Find the t and y coordinates of the vertex of the following function

$$y = -16t^2 + 48t + 6$$

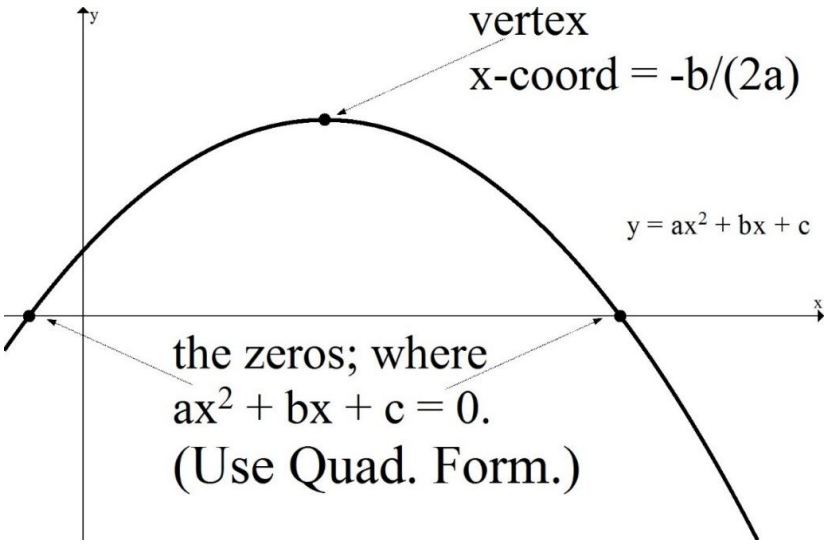
Click for [video clip of throwing a tennis ball]



Parabola Summary

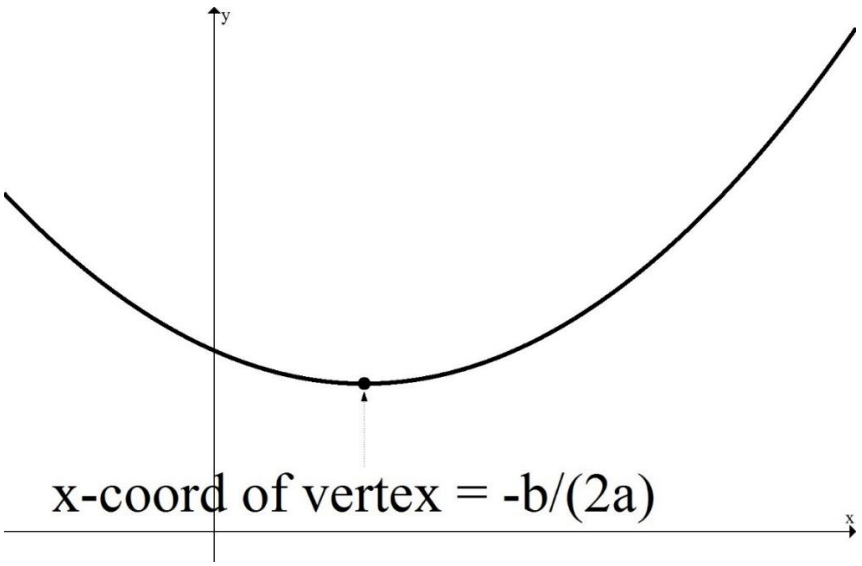
If *a* is negative, the parabola opens downward.

For example: $y = -5x^2 + 20x + 30.$



If *a* is positive, the parabola opens upward.

For example: $y = 2x^2 + 28x + 4.$



Note: A **quadratic equation** is an equation that can be written in the form

$$ax^2 + bx + c = 0.$$

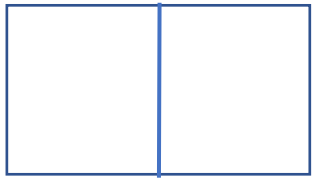
The solution(s) are given by the *quadratic formula*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Optimization Problems

- Draw/Label,
- What do we want?
- What are we given?

Example: You have 500 feet of fencing and you want to build a cage with one wall in the middle as shown.



How should you build it to maximize area?

Example: You run a petting zoo.

If you charge $8, you will sell 300 tickets/day giving a revenue of $2400 for that day.

If you charge $9, you will sell 280 tickets/day giving a revenue of $2520 for that day.

Assume tickets sold is a linear function of price, x.

Find the formula for tickets sold base on x.

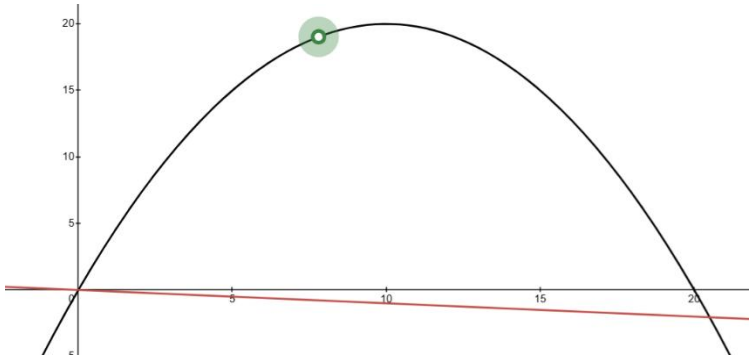
How many tickets should you sell to maximize revenue?

(*what is the max revenue? what is the price?)*

Example:

A ball follows the path $y = -\frac{1}{5}x^2 + 4x$.

In addition, the ground is sloping downward given by $y = -\frac{1}{10}x$. This is shown at right.



Link to desmos visual.

How would you find the following:

(a) The highest y-value of the ball.

(b) The location of the ball when it is farthest above the sloping ground.

(c) The location where the ball hits the ground.