Ch 2: Coordinates and Distances

Goal: Coordinates and the Distance Formula

Entry Task: Warm up questions for today

Label the following in the graph…

- origin, 1^{st}, 2^{nd}, 3^{rd}, and 4^{th} quadrants,
- negative and positive, x- and y-axes
- The point (x, y) = (3, -2)

What is the Pythagorean theorem?

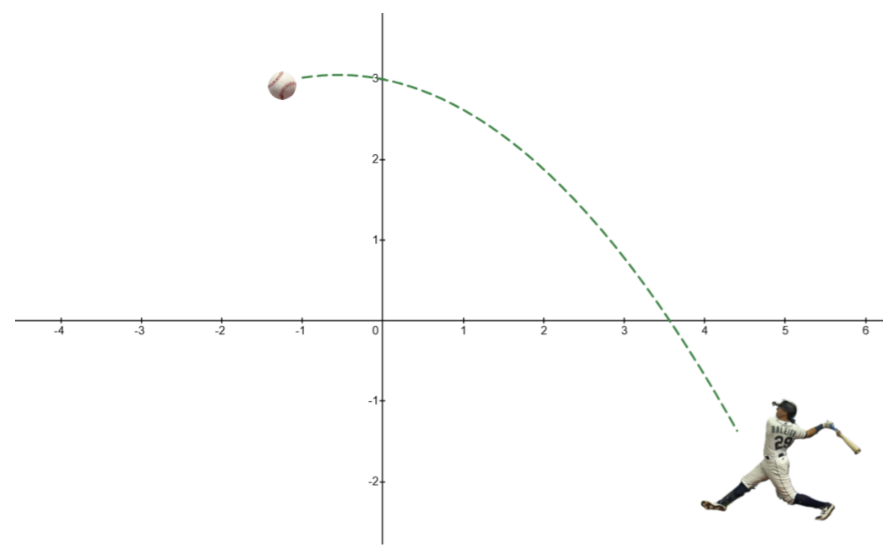
Another Unneccesary Math Joke

How did the math teacher get killed?

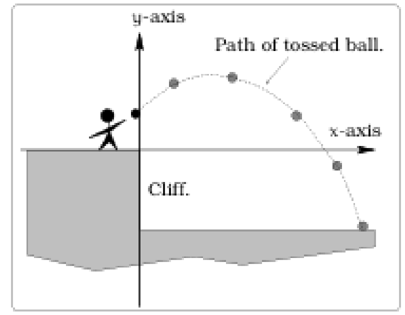
- … with axes.
- … it was a coordinated attack
- … they snuck up in ordered pairs.
- … the attackers escaped on a plane.

Chapter 1 & 2 Key Facts

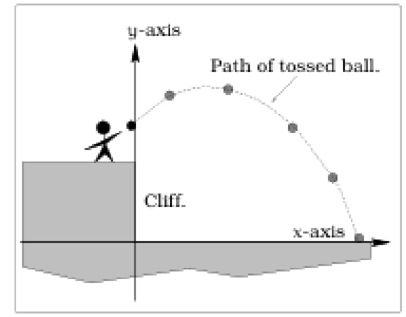
- $\text{speed} = \frac{\text{dist}}{\text{time}}$, $\text{rate} = \frac{\text{change in quantity}}{\text{change in time}}$
- $\text{dist} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- watch out for units!



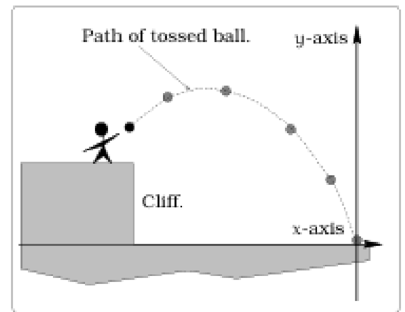
Discussion: Ideas on how to pick an origin?



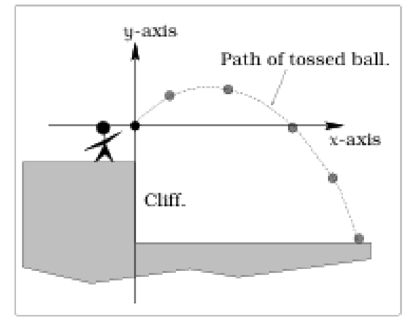
(a) Origin at the top of the ledge.



(b) Origin at the bottom of the ledge.

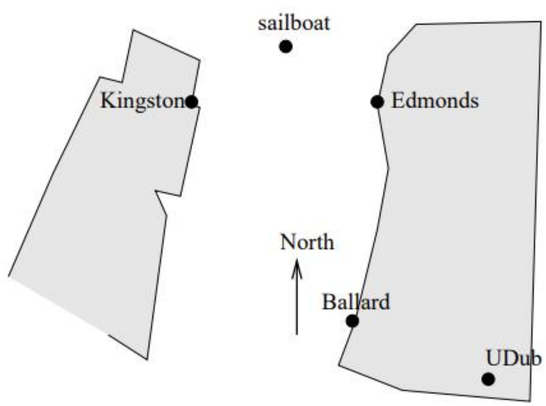


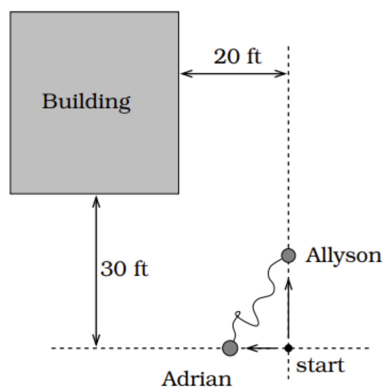
(c) Origin at the landing point.

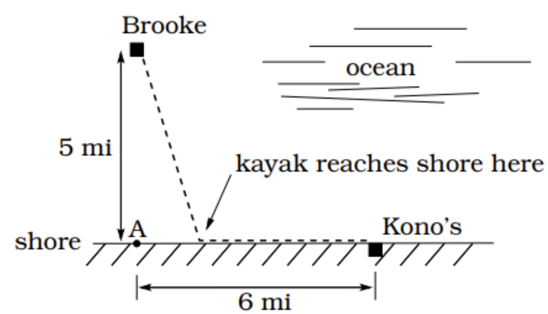


(d) Origin at the launch point.

Figure 2.7: Choices when imposing an xy-coordinate system.







Example: (HW 3.3a) Broken Water Main

Water flows from a broken main, forming a circular puddle whose radius grows at 2 in/min. An ant, 2 feet from the center, walks toward it at 5 in/min. When will it first touch the water?

Tip: "V.E.T.S." Problem Solving

1. **V**isualize: Draw axes, label unknowns.
2. **E**quations: Equations for curves.
3. **T**ranslate: Convert words to math.
4. **S**olve: Find intersections, distances, anything.

Distance

The distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$\text{dist} = \sqrt{(\mathrm{x}_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example:

What is the distance from $(4,3)$ and $(1, -1)$?

Derivation

Warning!!! *(The Sophomore's Dream)*

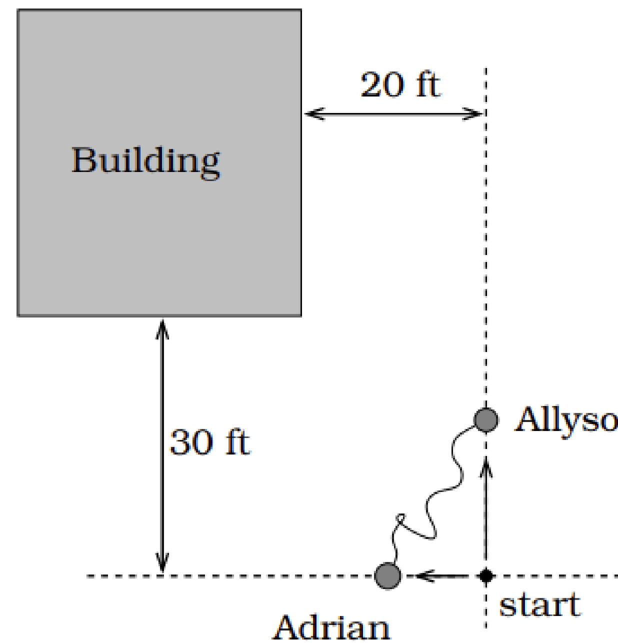
- $\sqrt{a^2 + b^2}$ does NOT simplify.
- It is NOT the same as $\sqrt{a^2} + \sqrt{b^2}$

(Here is review of when distribution is allowed.)

Example: (HW 2.6) Allyson and Adrian

Allyson and Adrian are connected at the ankles by a bungee cord. The cord is 30 feet long but can stretch to 90 feet. They start at the same location. Allyson runs north at 10 ft/sec and Adrian runs west at 8 ft/s

(a) Impose a coordinate system and label their locations after t seconds.
(b) When is the distance between them 30 feet?



Challenge: When will the bungee first touch the corner of the building?

Building Your Own Problem-Solving Routine

You should be starting to make a note sheet with your "problem-solving" routine. Here are two models I made up, but you should be building your own!

"Given-and-Want" Problem Solving

1. Identify what we **want**
2. Identify what we are **given**
3. **Fact finding**: Label Unknowns; Write related formulas
4. **Solve**

"V.E.T.S." Problem Solving

1. **V**isualize: Draw axes, label unknowns.
2. **E**quations: Equations for curves.
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Other notes:

- Check units at the start.
- Reread at the end, answer reasonable?

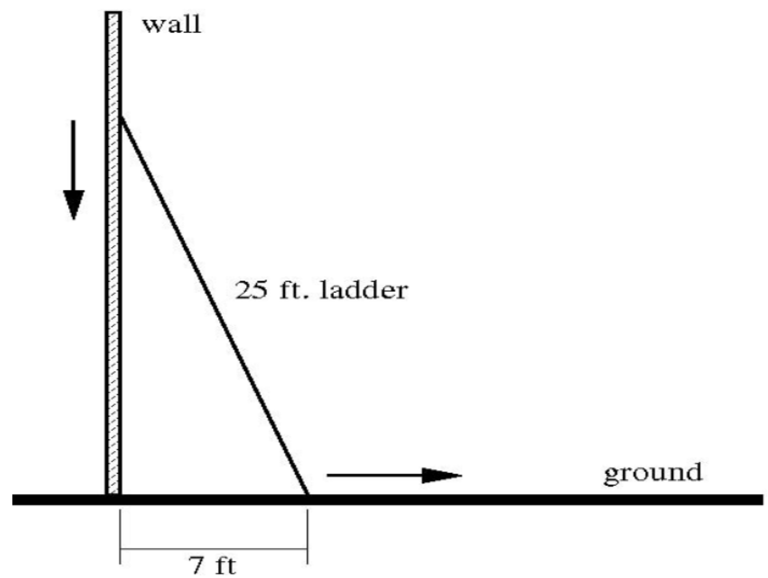
The following pages are not officially part of the lecture, they are parts of problems pulled directly from Math 124 Homework problems.

The skills we are practicing now are essential to being able to do applied calculus problems. Take a peek to get a preview of some of Math 124. Any ideas on how you might start labeling based on what we have done?

Math 124 Problem (in HW 2.1)

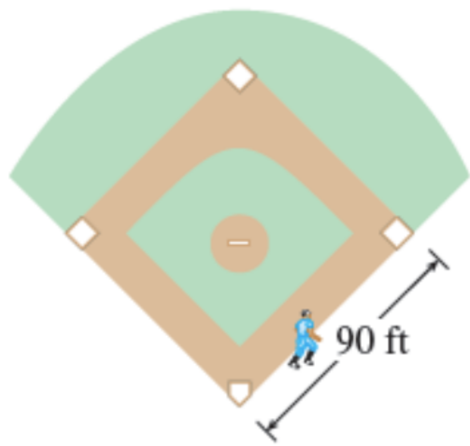
A ladder 25 feet long is leaning against the wall of a building. Initially, the foot of the ladder is 7 feet from the wall. The foot of the ladder begins to slide at a rate of 2 ft/sec, causing the top of the ladder to slide down the wall. The location of the foot of the ladder at time t seconds is given by the parametric equations $(7+2t,0)$.

(a) The location of the top of the ladder will be given by parametric equations $(0,y(t))$. The formula for $y(t)$=



Math 124 Problem (in HW 3.9)

A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s.

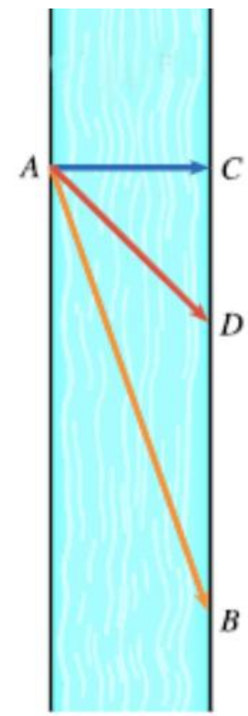


Find a formula for the distance from the runner to ***second base*** after t seconds.

Math 124 Problems (in HW 4.7)

10) A boat leaves a dock at 2:00 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3:00 PM How many minutes after 2:00 PM were the two boats closest together?

11) A man launches his boat from point A on a bank of a straight river, 2 km wide, and wants to reach point B, 2 km downstream on the opposite bank, as quickly as possible (see the figure below). He could row his boat directly across the river to point C and then run to B, or he could row directly to B, or he could row to some point D between C and B and then run to B. If he can row 6 km/h and run 8 km/h, where should he land to reach B as soon as possible? (We assume that the speed of the water is negligible compared to the speed at which the man rows.) (Hint: This question is based on EXAMPLE 4 in Section 4.7 of the textbook. However, for this question, the textbook has added a challenge which may require an unexpected solution. Look for it!)



14) A steel pipe is being carried down a hallway 9 ft wide. At the end of the hall there is a right-angled turn into a narrower hallway 6 ft wide. What is the length of the longest pipe that can be carried horizontally around the corner? (Round your answer one decimal place.)