Ch 20: Inverse Trig Functions

Goal: Define and use inverse trig functions.

Entry Task

Gavin's (broken) oven has a temperature that behaves sinusoidally.

- At *t=0*: temp is 425°F and rising
- First max 450°F at *t = 8.75* minutes
- First min 400°F at *t = 26.25* minutes

1. Find the sinusoidal model.
2. Sketch the graph for 0–70 minutes.
3. Consider the two questions:
   - What is the temperature at *t = 20 minutes*?
   - When is the temperature **440°F**?

- Temperature at $t = 20$ min:
  $T(20) \approx 414.15^{\circ}\mathrm{F}$
- Times in $[0, 70]$ when $T(t) = 440^{\circ}\mathrm{F}$:
  $t \approx 3.58,\ 13.92,\ 38.58,\ 48.92$ minutes

A motivating example

Find all solutions to $\sin(x) = \frac{1}{2}$.

Step 1: Draw y = sin(x) and the horizontal line y = ½

Step 2: Find the "principal solution" (closest to phase shift)

Step 3: Find the "symmetric solution" (next closest)

Step 4: Use the period to summarize the rest.

The ***inverse trig functions*** are defined to give the "principal solution"

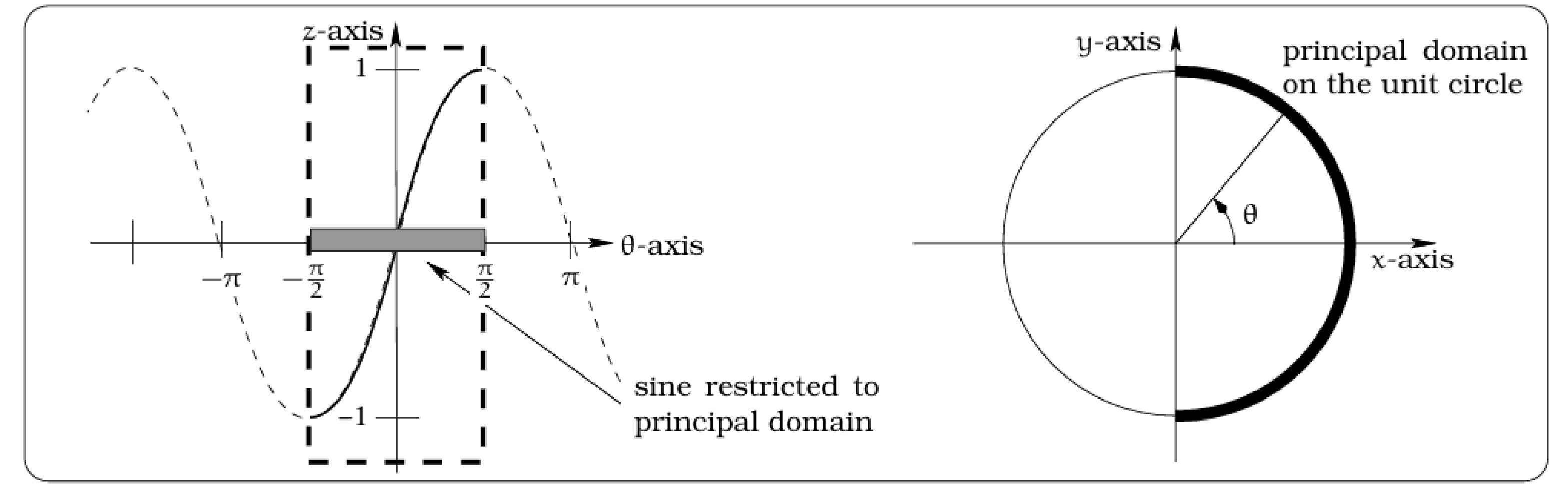

Figure 20.5: Principal domain for sin(θ).

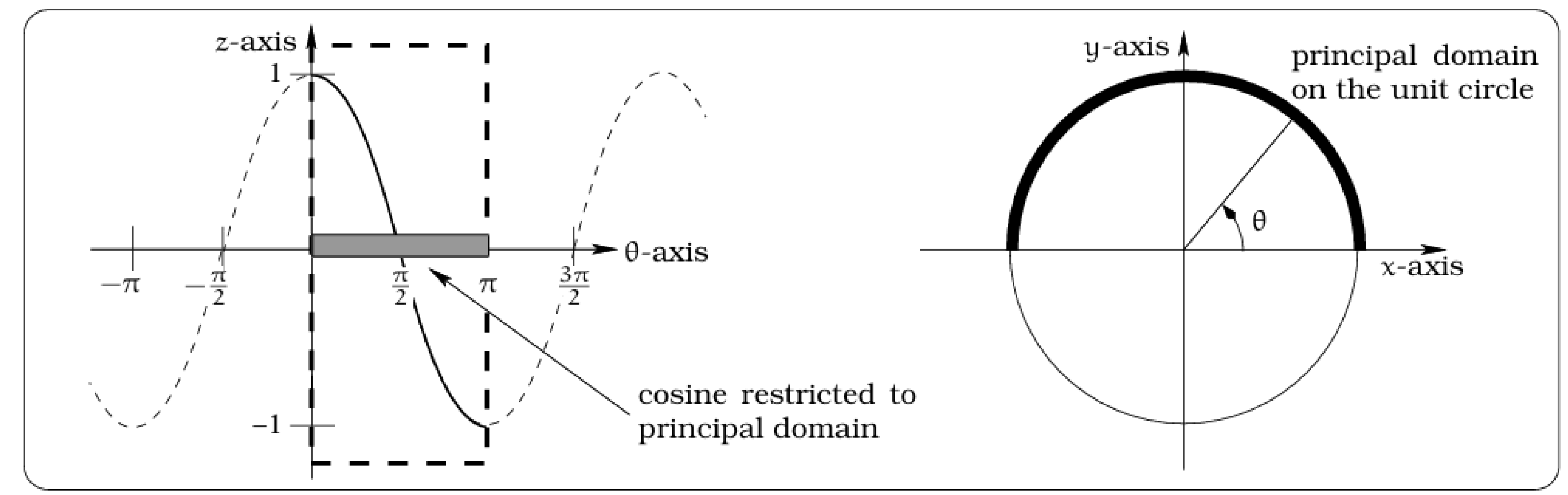

Figure 20.6: Principal domain for cos(θ).

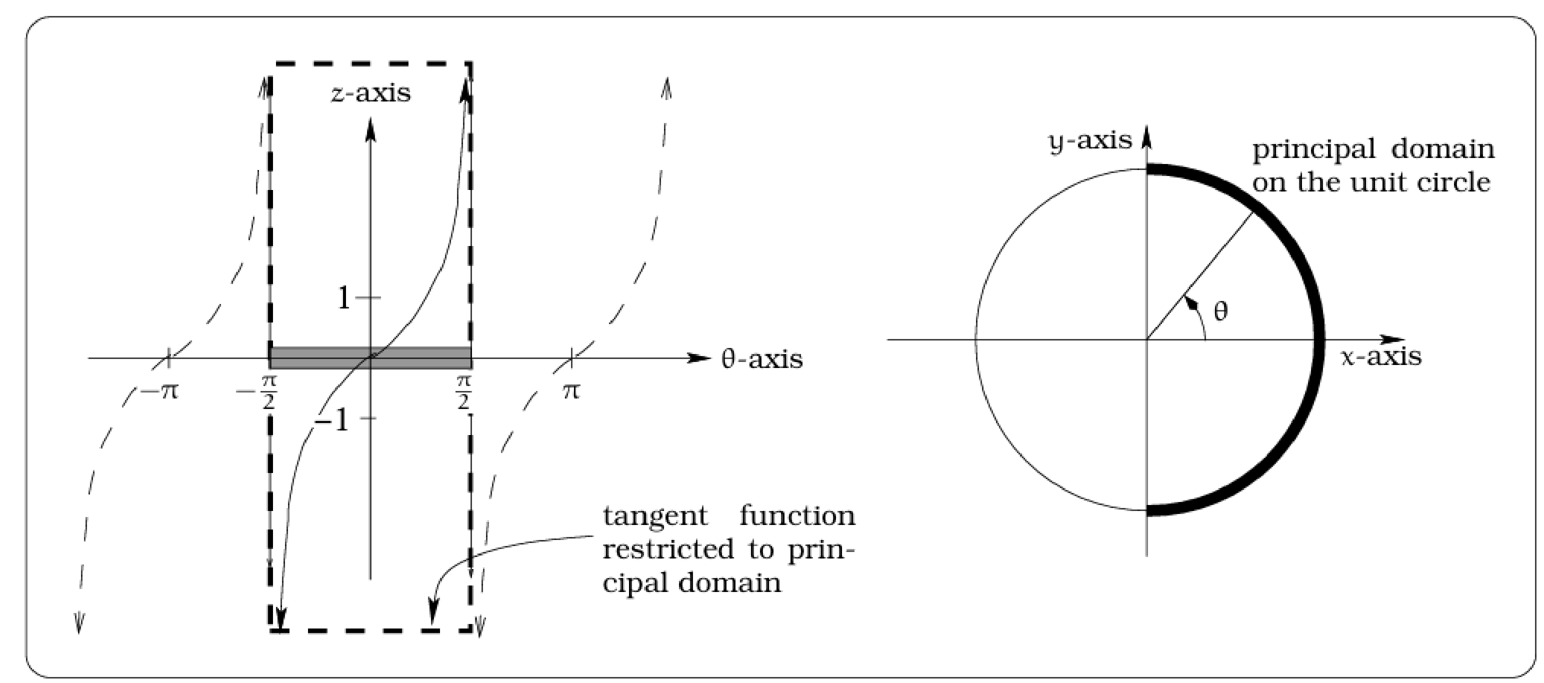

Figure 20.7: Principal domain for tan(θ).

1.	$3\tan(x) = 4$	→	$x \approx 0.9273$
2.	$4\cos(2x - 1) = 1$	→	$x \approx 1.1591$
3.	$\sin(x) = 3$	→	no solution
4.	e^(tan(x^3)) = 10	→	$x \approx 1.0510$
5.	$\tan(\sin(\cos(x))) = 0.2$	→	$x \approx 1.3708$
6.	$\sin^2(x) = 0.09$	→	$x \approx 0.3047$

Solving Mechanics: *Using your calculator, find the principal solution of the following equations (in radians):*

1. $3\tan(x) = 4$

2. $4\cos(2x - 1) = 1$

3. $\sin(x) = 3$

4. $e^{\tan(x^3)} = 10$

5. $\tan(\sin(\cos(x))) = 0.2$

6. $\sin^2(x) = 0.09$

Okay, now how do we find other solutions?

Option 1: Graphical Reasoning Method

1. Find the principal solution.
2. Sketch a graph and find the nearest max/min.
3. Reflect horizontally for the symmetric solution.
4. Add the period for all solutions.

Example: Solve $3\sin\left(\frac{2\pi}{35}(t-10)\right)+4=5$.

First few solutions:

$t \approx 11.89,\ 25.61,\ 46.89,\ 60.61\ (\text{minutes})$

You try: Give *all* solutions to sin(x) = 0.6.

Option 2: Algebraic Method

Step	***Sine case***	***Cosine case***	***Tangent case***
1. Find principal solution	$\theta = \sin^{-1}(c)$	$\theta = \cos^{-1}(c)$	$\theta = \tan^{-1}(c)$
2. Find symmetry solution	$\theta = -\sin^{-1}(c) + \pi$	$\theta = -\cos^{-1}(c)$	*not applicable*
3. Write out multiples of period $k = 0, \pm 1, \pm 2, \cdots$	$2k\pi$	$2k\pi$	$k\pi$
4. Obtain general principal solutions	$\theta = \sin^{-1}(c) + 2k\pi$	$\theta = \cos^{-1}(c) + 2k\pi$	$\theta = \tan^{-1}(c) + k\pi$
5. Obtain general symmetry solutions	$\theta = -\sin^{-1}(c) + \pi + 2k\pi$	$\theta = -\cos^{-1}(c) + 2k\pi$	*not applicable*

All solutions:

$$x \approx 0.6435 + 2\pi k \quad \text{or} \quad x \approx 2.4981 + 2\pi k, \quad k \in \mathbb{Z}$$

Full applied example: A weight hanging from a spring moves sinusoidally.

Lowest Point: 15 cm at t = 5 seconds.

Highest Point: 37 cm at t = 9.4 seconds.

During the first 20 seconds, how much time is the weight above 28 cm?

During the first 20 seconds, time above 28 cm:

- Crossings with 28 cm in $[0, 20]$:
  $t \approx 2.54,\ 7.46,\ 11.34,\ 16.26$ s
- Above 28 cm on intervals:
  $[0, 2.54],\ (7.46, 11.34),\ (16.26, 20]$

Total time above 28 cm:

$$\boxed{\approx 10.18 \text{ seconds}}$$

Another Example: Given $y = 22\sin\left(\frac{2\pi}{18}(t-3)\right) + 46$.

Find the first four positive values of t when this function equals 50.

First four positive solutions:

$$\boxed{t \approx 3.52,\ 11.48,\ 21.52,\ 29.48}$$