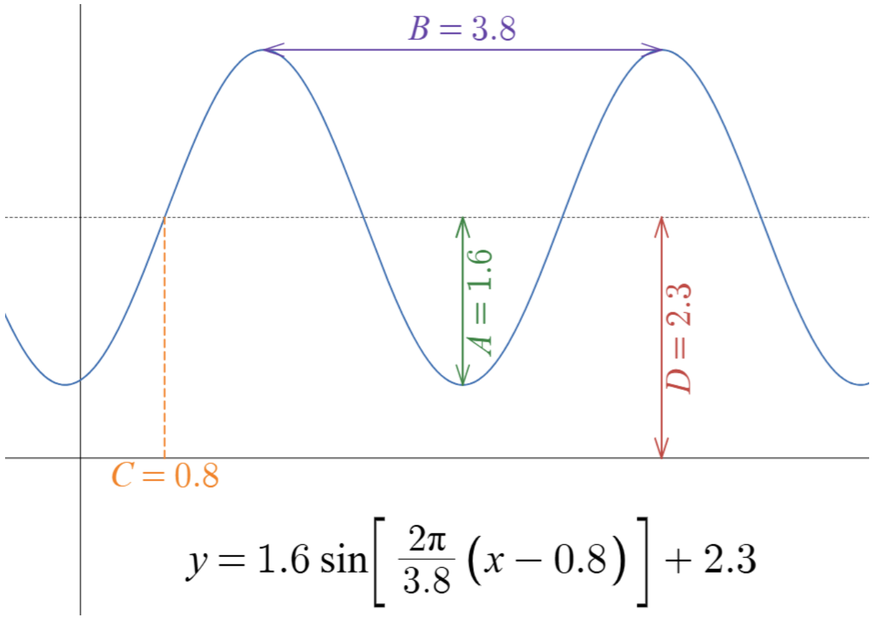
Ch 19: Sinusoidal Modeling

Goal: Modeling with sine waves. We will explore the parameters A, B, C and D for the model below.

The general **sinusoidal model** is given by

$$y = A \sin\left[\frac{2\pi}{B}(x - C)\right] + D$$

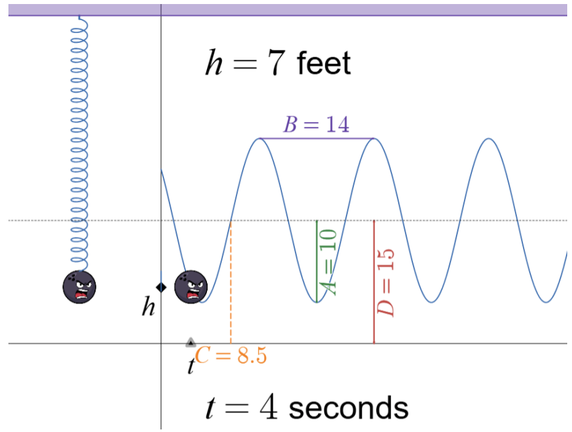


The graph above is a sneak peek at what these values.

And here is an interactive visual: https://www.desmos.com/calculator/kj5x696jbp

Motivation: There are a large number of applications, here are a few you will encounter regularly in your life including sound, light, mechanical, & signal processing.

Click here for a visual of a mechanical oscillation that you will see as an example in math and physics.



Finding D – *the "mean" value of the wave*

$$y = \sin(\mathrm{x}) + D$$

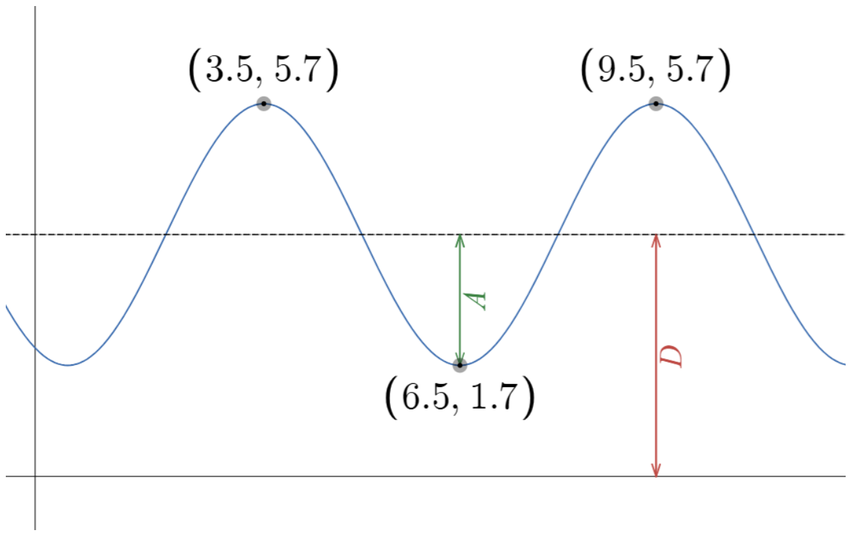
Q: What does "D" to the graph of y = sin(x)?

Finding A – *the "amplitude" of the wave*

$$y = A\sin(\mathrm{x})$$

Q: What does "A" do to the graph of y = sin(x)?

Q: How would you find D in the following graph?



Q: How would you find D in the following graph?

Finding B – *the "period" of the wave*

$$y = \sin\left[\frac{2\pi}{B}\mathrm{x}\right]$$

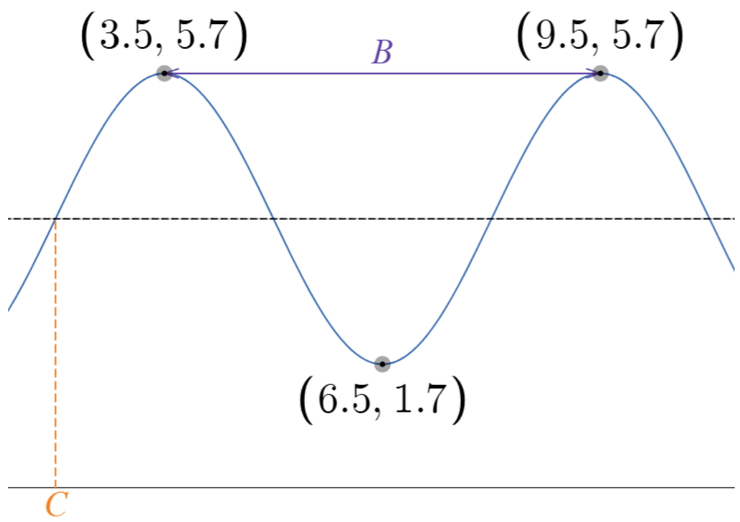
Q: What does "B" to the graph of y = sin(x)?

Finding C – *the "phase shift" of the wave*

$$y = \sin(x - C)$$

Q: What does "C" do to the graph of y = sin(x)?

Q: How would you find B in the following graph?



Q: How would you find C in the following graph?

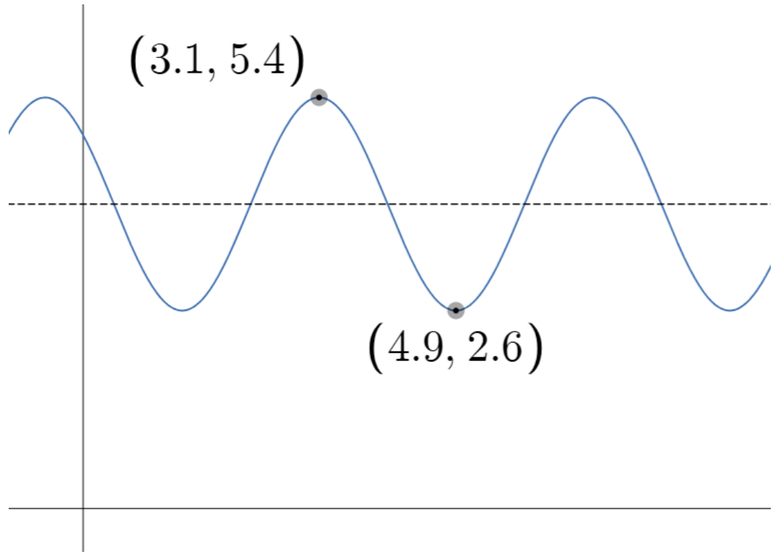
Summary: In the general sinusoidal model

$$y = A \sin\left[\frac{2\pi}{B}(x - C)\right] + D$$

- A = amplitude $= \dfrac{\text{max output} - \text{min output}}{2}$
- D = mean $= \dfrac{\text{max output} + \text{min output}}{2}$
- B = period = horiz. dist from peak-to-peak
- C = phase shift = x-coordinate of a peak $- \dfrac{B}{4}$

You try:

Find the model given that the function has a peak at (3.1, 5.4) then a valley at (4.9, 2.6).



Tip: When reading homework and doing exam questions, I encourage you to think about the picture first!

Drawing a given wave

Example: Sketch a graph of $y = 2\sin\left[\frac{2\pi}{3}(x-2)\right] + 5$

Standard form note:
Recall, the order of transformations matters. If you want to use this model in this form, you must write it in this form.

Example: Sketch a graph of $y = 4(\sin(2x\text{-}6) + 5)$

Other notes that might be of value of you.

Phase shift:
There are an infinite number of choices for the phase shift. It can be any value of x where the graph crosses the mean line and is increasing.

Wavelength:
The period is sometimes called the wavelength, they mean the same thing.

Period over 4 Trick:
The graph goes from mean line to peak to mean line to valley to mean line in a full wave cycle.
Each transition is a fourth of a period.

Frequency:
The inverse of the period is called the frequency which gives the number of waves per second.