Ch 17: The Trig and Circular Functions

Goal: Learn how to find the (x,y) coordinates on a circle given an angle and radius.

Consider a right triangle with one angle θ.

We give the following names to the six ratios of the sides…

Sine	$sin(\theta)$	
Cosine	$cos(\theta)$	
Tangent	$tan(\theta)$	
Cosecant	$csc(\theta)$	
Secant	$sec(\theta)$	
Cotangent	$cot(\theta)$	

Some Nice Triangles and Angles
(45-45-90, 30-60-90)

Easy ways to remember (0-1-2-3-4 method)

Calculator Mode warning

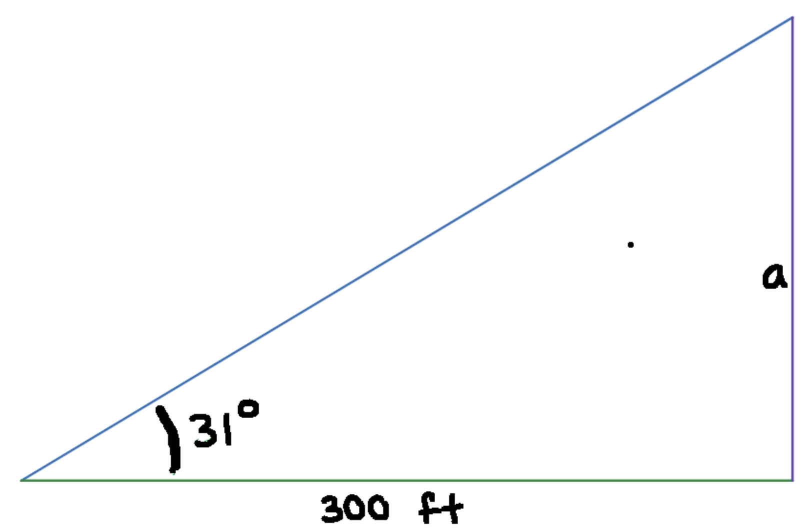
Get your calculator out and compute…

$$sin\left(\frac{\pi}{6}\right) \quad and \quad sin(30)$$

Trig Problems

Example:

A right triangle has an angle of 31 degrees. The side adjacent to that angle has length 300 ft. What is the length of the side opposite that angle?

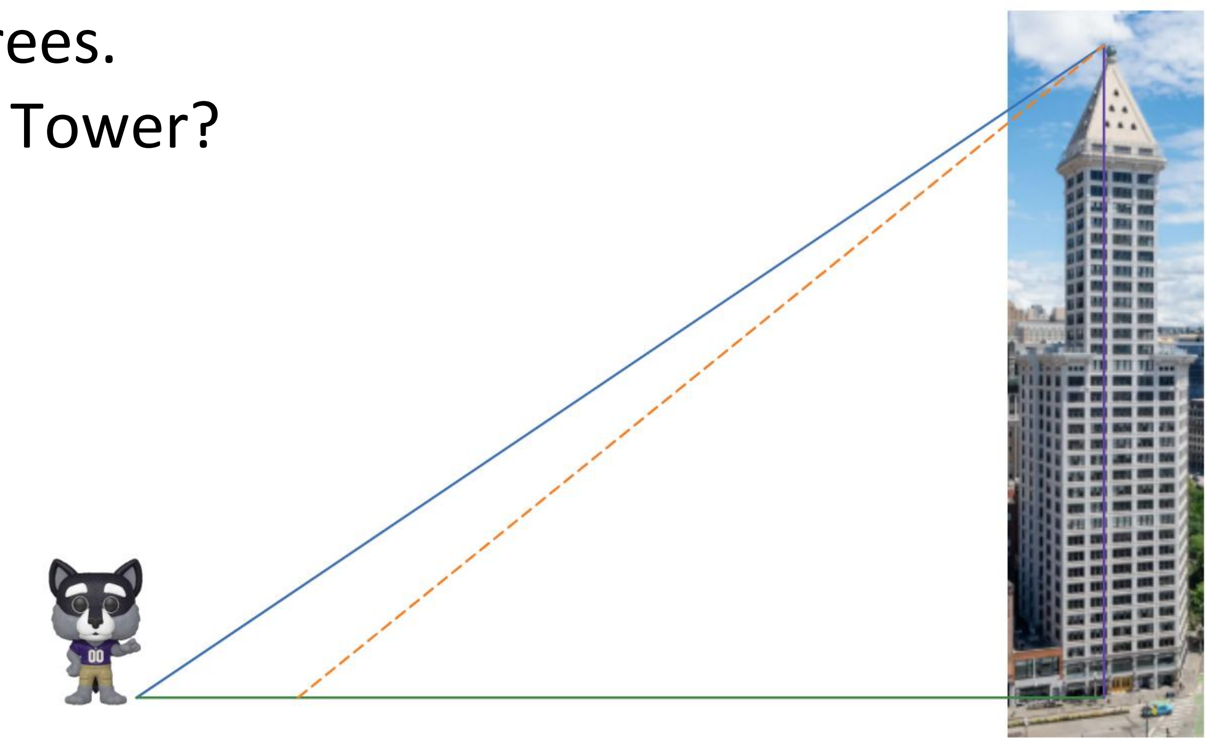


Example:

You are looking up at the Smith Tower.

- From your location the top is 34 degrees above horizontal.
- You walk forward 110 feet. Then you measure the angle to the top is 39 degrees.

How tall is the Smith Tower?



Trig Tips:

- Label the side(s) and angle(s) you are ***given*** and ***want***.
- You likely will write an equation for every right triangle.
  - Combine and solve!
- There is often more than one way to do it!

Locations on a circle

Example: ***Motivation***

Harry is standing on the easternmost edge of a circular track with radius 40 feet. Harry rotates 10 degrees per second counterclockwise.

- Where will be in 3 seconds?
- Where will be in 5 seconds?
- Where will be in 9 seconds?
- Where will be in 1 minute?

Observations…

Important Note:

- Sine and Cosine are defined to give the correct value on unit circle ***beyond*** 90 degrees. This extends the definition of trig functions.

General Facts

If an object is on a circle of radius r with center at the origin and its angle in standard position is θ, then the (x, y) location of the object is given by

- $x = r \ \cos(\theta)$
- $y = r \ \sin(\theta)$

The challenge in practice is finding the angle, which involves

- Finding the starting location angle = θ_0
- Finding angular speed = ω

Then using change in angle = ωt

and computing:

$$\theta = \omega t + \theta_0$$

If the center of the circle is at (x_c, y_c), then we the only change is…

- $x = x_c + r\cos(\theta)$
- $y = y_c + r\sin(\theta)$

And the most general version is…

- $x = x_c + r\cos(\omega t + \theta_0)$
- $y = y_c + r\sin(\omega t + \theta_0)$

Example: Ron starts at the southmost edge of a merry-go-round of radius 8 feet and is rotating at 2 rad/sec. Assume the center of the merry-go-round is the origin and south is the negative y-axis.

(a) Where will Ron be located in 3 seconds?

(b) Find the general formula for Ron's location, $(x(t),y(t))$, in t seconds.

Some comments on slopes of radial and tangent lines