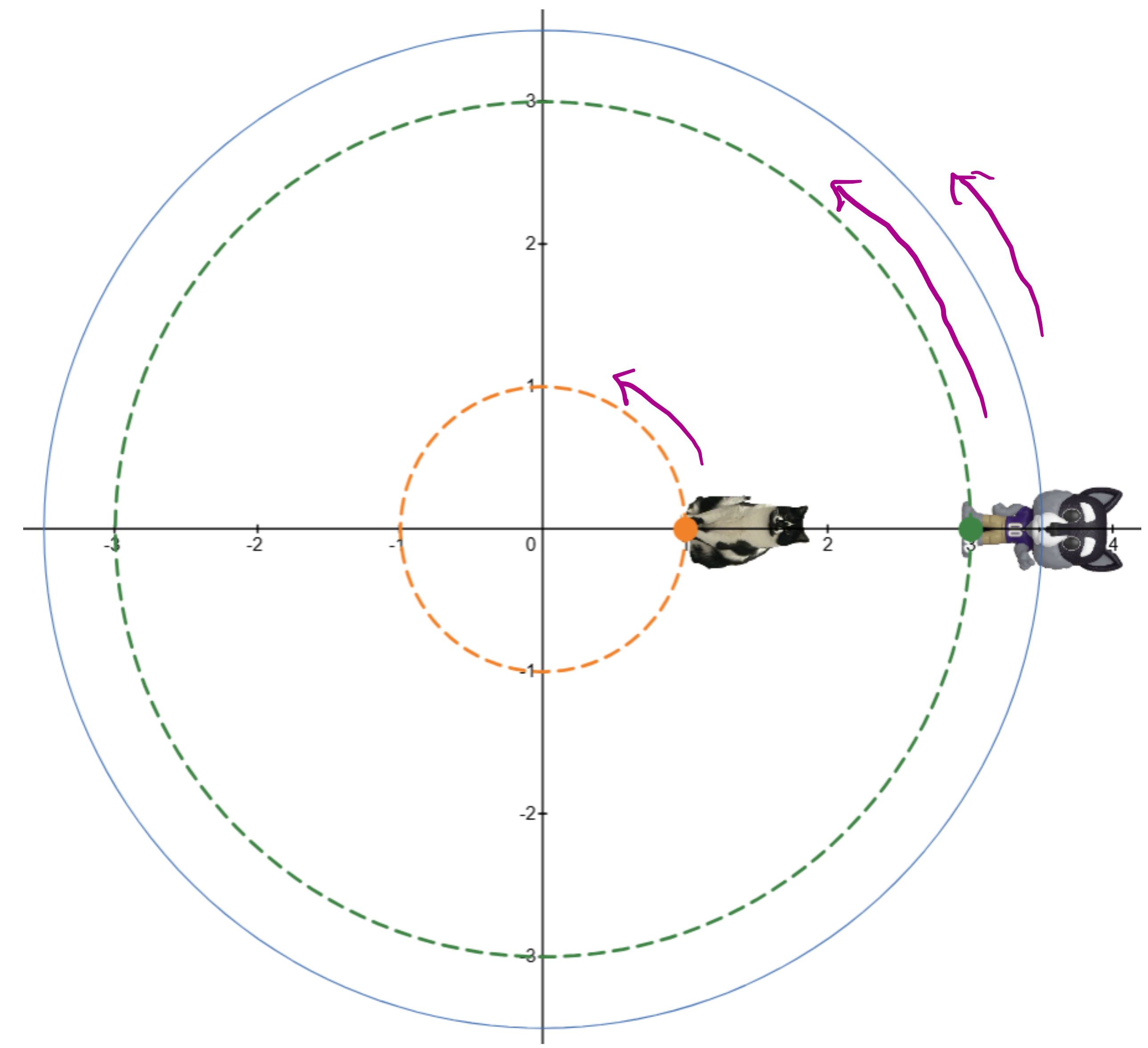
Ch 16: Measuring Circular Motion
Goal: Three key formulas and ideas for circular motion

Motivation / Entry Task
Consider two things on a merry-go-round as shown.

- Penelope (the cat) is 1 foot from the center
- Butch is 3 feet from the center.
- Both complete one rotation in 14 seconds.

Q: Who is “traveling” faster?
Q: Who is “rotating” faster?



Animation: https://www.desmos.com/calculator/cr3jyzibfu

Fundamental Observation:

- The radius matters! You can’t go from linear speed to angular speed (or vice versa) without the radius.

Two ways to measure circular motion:

1. **Linear speed**: $\mathrm{v} = \frac{\mathrm{d}}{\mathrm{t}}$
   - $\text{linear speed} = \frac{\text{distance}}{\text{time}}$
   - unit examples:

2. **Angular speed**: $\omega = \frac{\theta}{\mathrm{t}}$
   - $\text{angular speed} = \frac{\text{angle}}{\text{time}}$
   - unit examples:

Conversion Notes:

- $1 \text{ revolution angle} = 360° = 2\pi \text{ radians}$
- $1 \text{ revolution dist} = \text{circumference} = 2\pi r$

Example: The wheels of a car are turning at 200 RPM.

1. Convert to radians/min and degree/min.

2. If the radius of the tire is 3 feet, find the linear speed of the edge of the tire (note: this is the same as speed of the car).

Summary and Key Formulas

Given an object going around circle at a constant speed, we label

- $\mathrm{r} = \text{radius} = \text{distance of object from the center}$
- $\theta = \text{angle rotated}$
- $\mathrm{t} = \text{time}$

and we label and compute…

- $\text{arc length} \quad = \mathrm{s} = \theta \mathrm{r}$
- $\text{linear speed} \quad = \mathrm{v} = \frac{\mathrm{s}}{\mathrm{t}}$
- $\text{angular speed} \quad = \omega = \frac{\theta}{\mathrm{t}}$

Q: One of these formulas MUST have θ in radians.
Do you remember which one?

The fundamental formula relating linear and angular speed can be obtained by putting these equations together, try it…

Side Comment on the "Name" of this Formula:
This formula doesn't have a formal math name as far as I know, so I am taking suggestions.

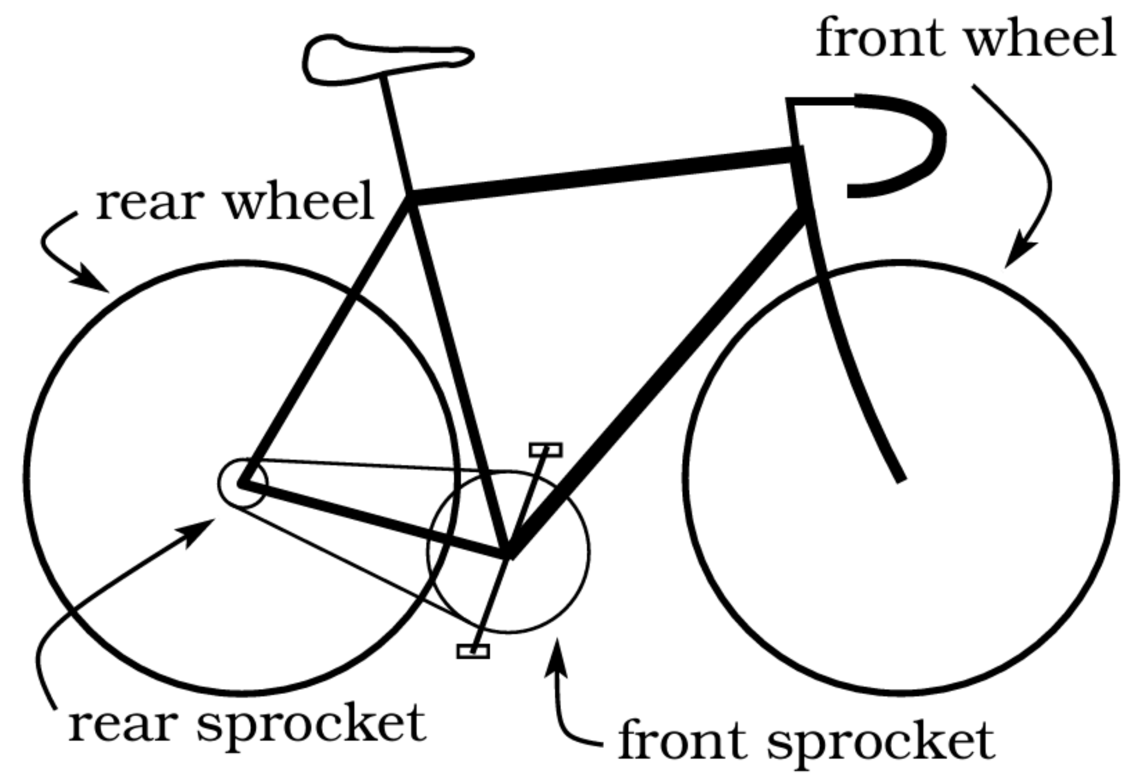
Here are some of my bad naming attempts:

"Spin-to-Speed Converter" *"The Angulinear Speed Formula"*
"Speed Exchange Program" *"Wheelationship Formula"*
"You spin me right round formula" *"Round-round get around formula"*

Engineering/Physics names:

"Tangential Velocity Formula" *"Linear-Angular Velocity Relationship"*
"No-Slip Condition" *"Gear Ratio Law"*

Quick Example:
A bicycle is moving at 30 mph. A reflector in the tire spokes makes 5 revolutions per second.
What is the radius of the wheel?



Belt and Wheel Problem Key Facts:

- Same center (or axle) implies same angular speed.
- Connected by a belt (or chain) implies same linear speed.
- If using radians, then $v = wr$.

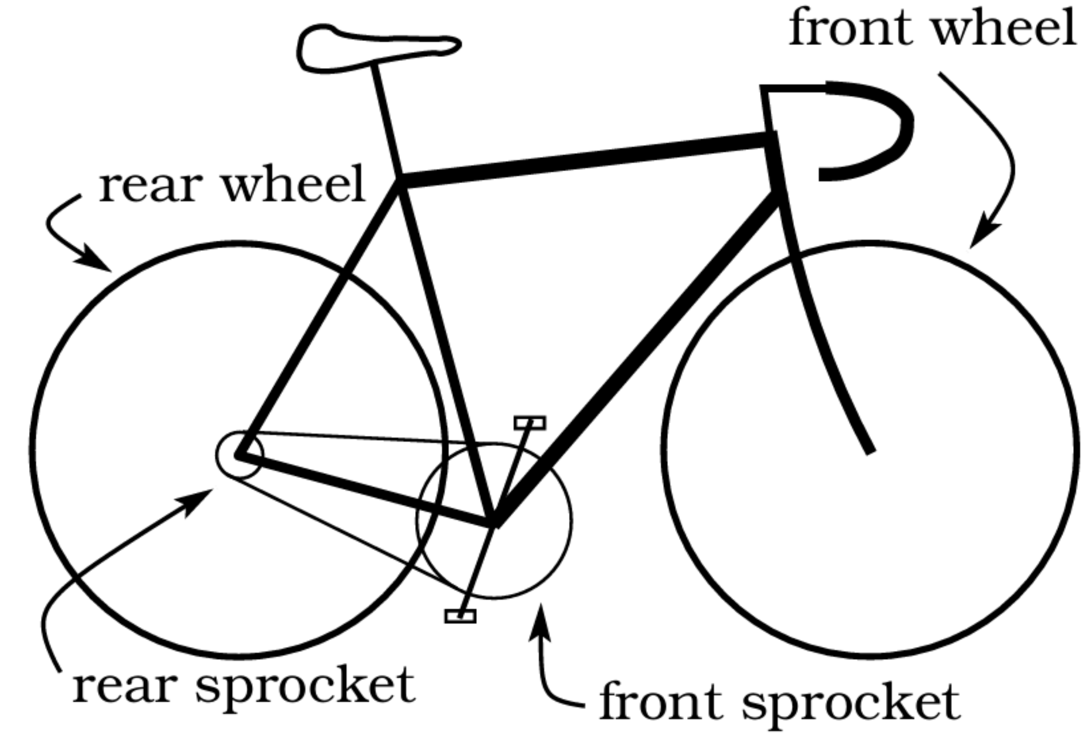
Example: For the bicycle shown…

- radius of the wheels = 30 inches
- radius of the rear sprocket = 4 inches
- radius of the front sprocket = 8 inches

How many revolutions per minute of the pedals will make the bike go 35 mph (36960 in/min)?

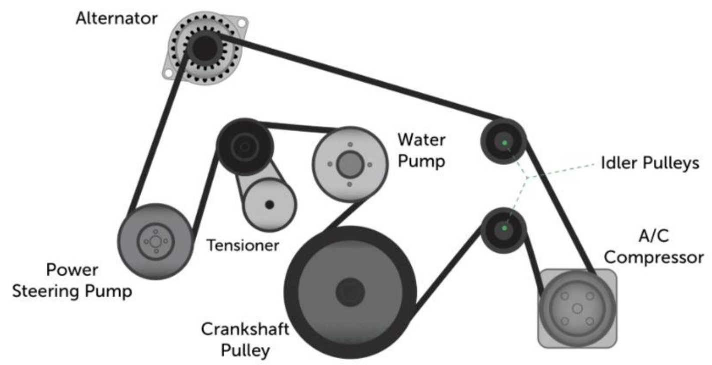
Belt and Wheel Problem Key Facts *(again)*:

- Same center (or axle) implies same angular speed.
- Connected by a belt (or chain) implies same linear speed.
- If using radians, then $v = wr$.

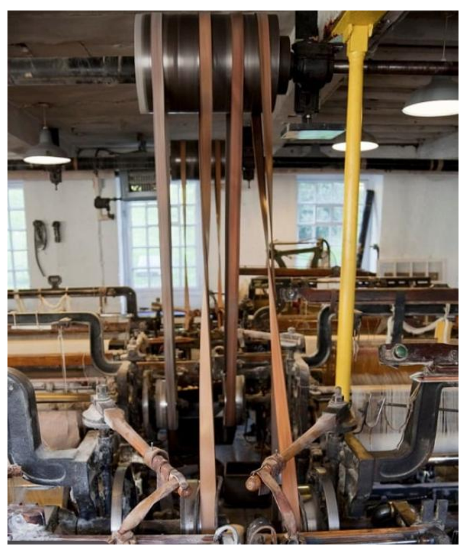


Some belt and wheel systems you may have seen besides a bike:

A car

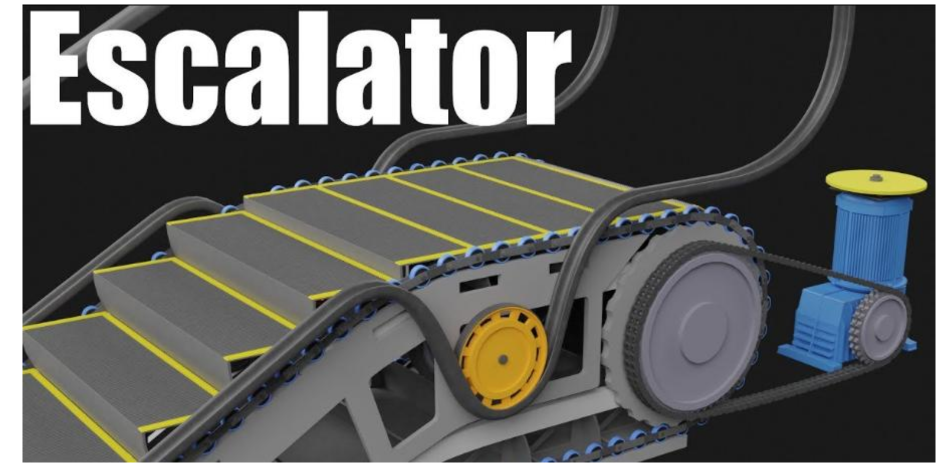


A factory

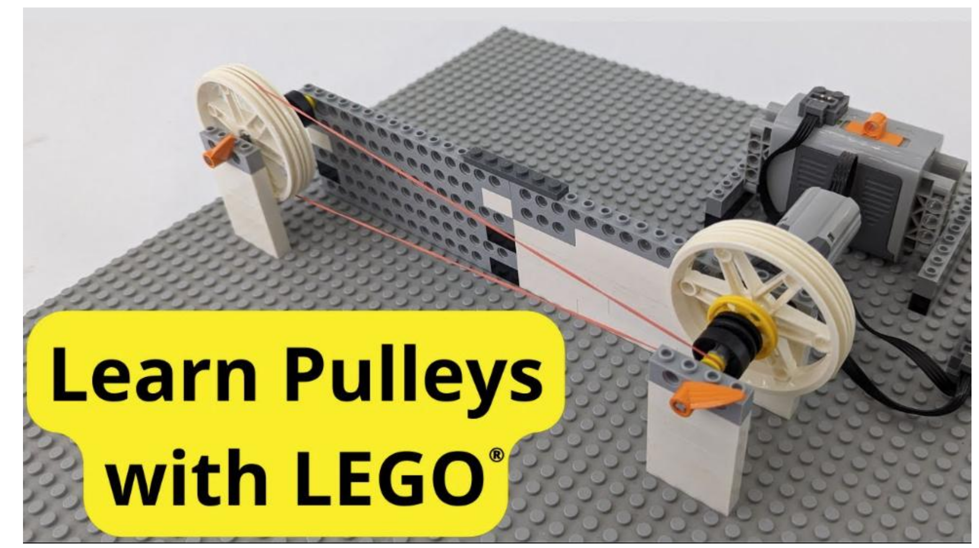


Inside the "Big Ben" clock

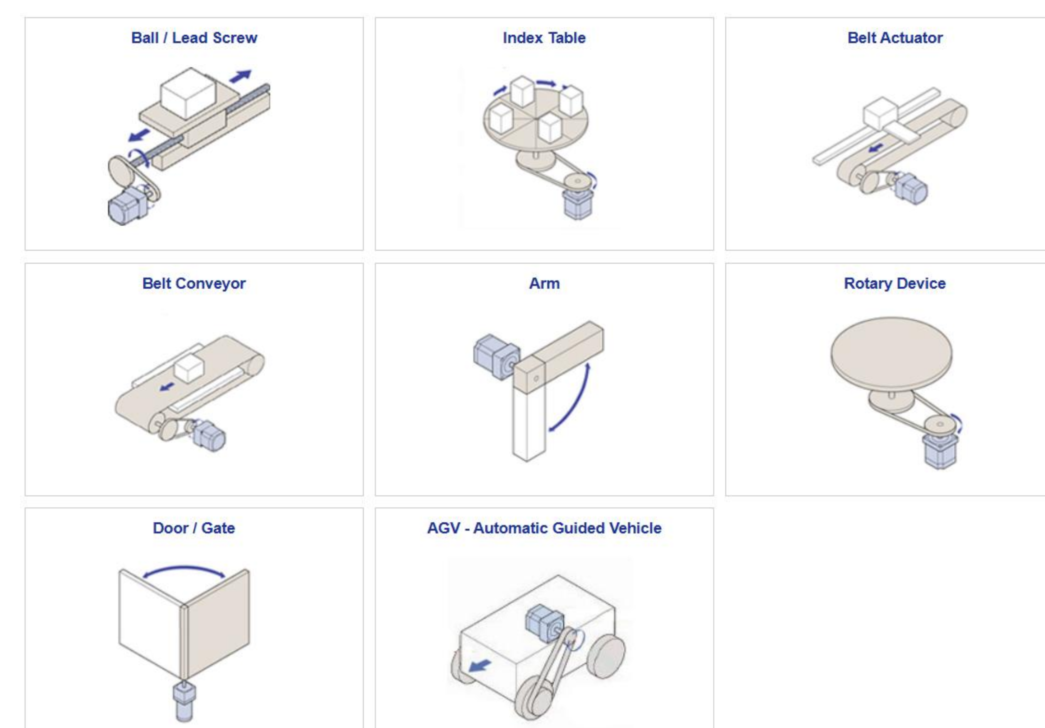
An Escalator



Legos



From a motor sales website:



Another example:

You are given the following information:

A: r = 10 in, v = 10,000 in/min

B: r = 2 in

C:

D: r = 3 in

E: r = 10 in, w = 20,000 rad/min

What is the radius of C?