Ch 14: Rational Functions
Goal: An intro to rational functions and asymptotes. But first…

Entry Task: *Some review notes*

1. Give the form of the following models:
   a. Lines:

   b. Quadratic:

   c. Exponentials:

2. Sketch rough, quick graphs of
   $y = x$

   $y = x^2$

   $y = e^x$

3. Use any skills you have (including ch 13) to sketch the graphs:

   $y = 4(x + 3) + 5$

   $y = -2(x - 1)^2 + 5$

   $y = e^{-x} + 3$

Background

A ***polynomial*** looks like this

$$y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

The largest power, *n*, is called the ***degree.***

Examples:

New today

A **rational** function is a fraction (i.e. "ratio") of two polynomials. Meaning it looks like

$$y = \frac{p(x)}{q(x)},$$

where $p(x)$ and $q(x)$ are polynomials.

Examples:

The study of polynomial and there uses is a massive area of math with historic problems of deep importance and applications in many branches of science.

Example: Plug in values and plot points to get a graph of

$$f(x) = \frac{1}{x}$$

Fraction Observations/Review

1. Consider $y = \frac{5}{B}$

   Tell me everything you can about when B is large (and positive or negative) or close to zero.

2. Consider the fraction $y = \frac{x-2}{x^2-25}$

   Find the value (*or values*) that make this equal to zero.

<u>Linear-to-linear rational functions</u>

A **linear-to-linear** rational function is one where the numerator and denominator are both linear (i.e. degree 1).
We write them in the form…

$$y = \frac{ax + b}{x + c}.$$

Examples:

Examples:

- Zeros (i.e. roots): Where is the numerator zero?
- Vertical Asymptote: Where is the denominator zero?
- Horizontal Asymptote: What happens when x gets large?

<u>Three important properties</u>

Summary

Linear-to-linear rational functions are of the form

$$y = \frac{ax + b}{x + c},$$

where

- $x = -c$ is a vertical asymptote
- $y = a$ is a horizontal asymptote
- $x = -\frac{b}{a}$ is the only zero.

Graph to have in mind:

Solving for the Constants (same idea as our other models!)

a. Plug in data
b. Combine and solve for a, b and c

How will you start these chapter 14 homework problems?

For each of the following, find the linear to linear function $f(x)$ satisfying the given requirements.

(a) $f(0) = 0$, $f(10) = 5$, $f(20) = 8$

$f(x) =$ ____

(b) $f(0) = 20$, $f(5) = 5$, $f(20) = 3$

$f(x) =$ ____

(c) $f(10) = 16$, $f(30) = 18$, and the graph of $f(x)$ has $y = 20$ as its horizontal asymptote

$f(x) =$ ____