

Ch 12: Logarithms

Goal: Learn the inverse of $y = e^x$ and more generally $y = b^x$

Entry Task: (Fundamentals and Calculator work)

Simplify or compute:

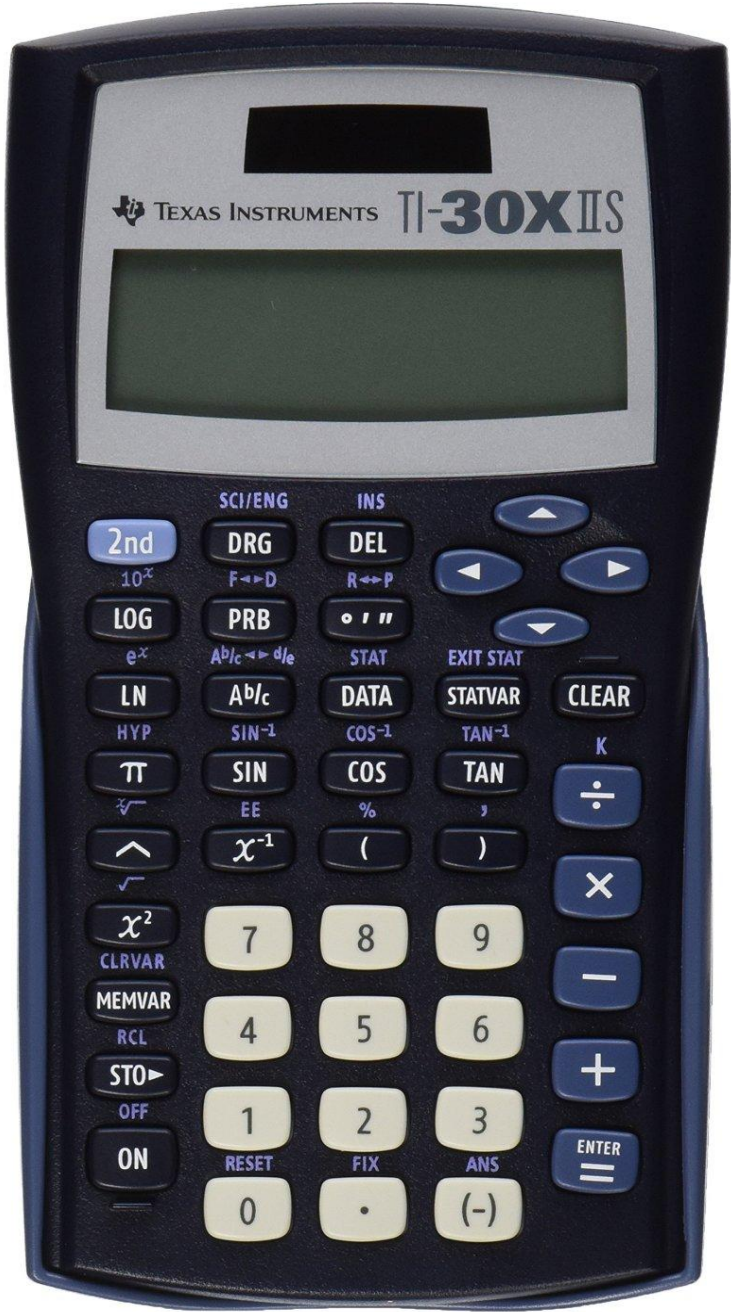
$5^3 =$	$125^{1/3} =$
$\ln(1) =$	$e^0 =$
$\log_{10}(1000) =$	$10^3 =$
$\log_2(256) =$	$2^8 =$
$\ln(e^{10}) =$	$e^{\ln(3)} =$

Compute in your calculator:

$\ln(2^3) =$	$3 \ln(2) =$
$\ln(2 \cdot 3) =$	$\ln(2) + \ln(3) =$
$\log_{10}(300) =$	$\frac{\ln(300)}{\ln(10)} =$

The inverse of $y = e^x$ is $\ln(y) = x$.
The inverse of $y = b^x$ is $\log_b(y) = x$.
Important facts:

- $\ln(a^b) = b \ln(a)$
- $\ln(ab) = \ln(a) + \ln(b)$
- $\log_b(y) = \frac{\ln(y)}{\ln(b)}$



The inverse of an exponential

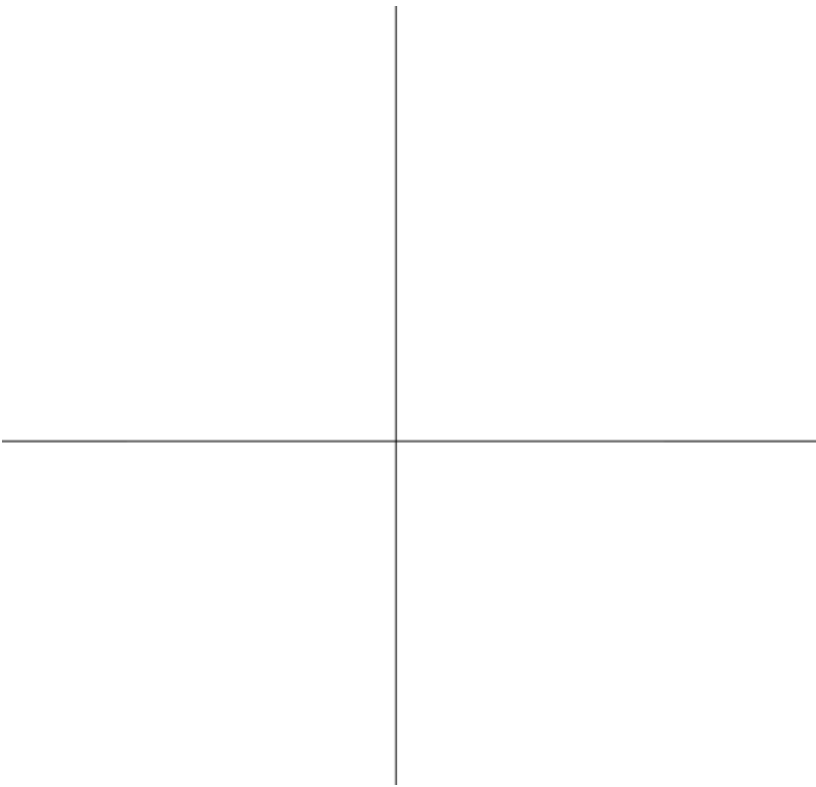
The inverse of $f(x) = e^x$ is $f^{-1}(x) = \ln(x)$.
The inverse of $g(x) = b^x$ is $g^{-1}(x) = \log_b(x)$.

General properties

- *Cancellation:* $\ln(e^x) = x$ and $(e^{\ln(x)}) = x$
 $\log_b(b^x) = x$ and $(b^{\log_b(x)}) = x$

- *Domain/Range/Graphs:*

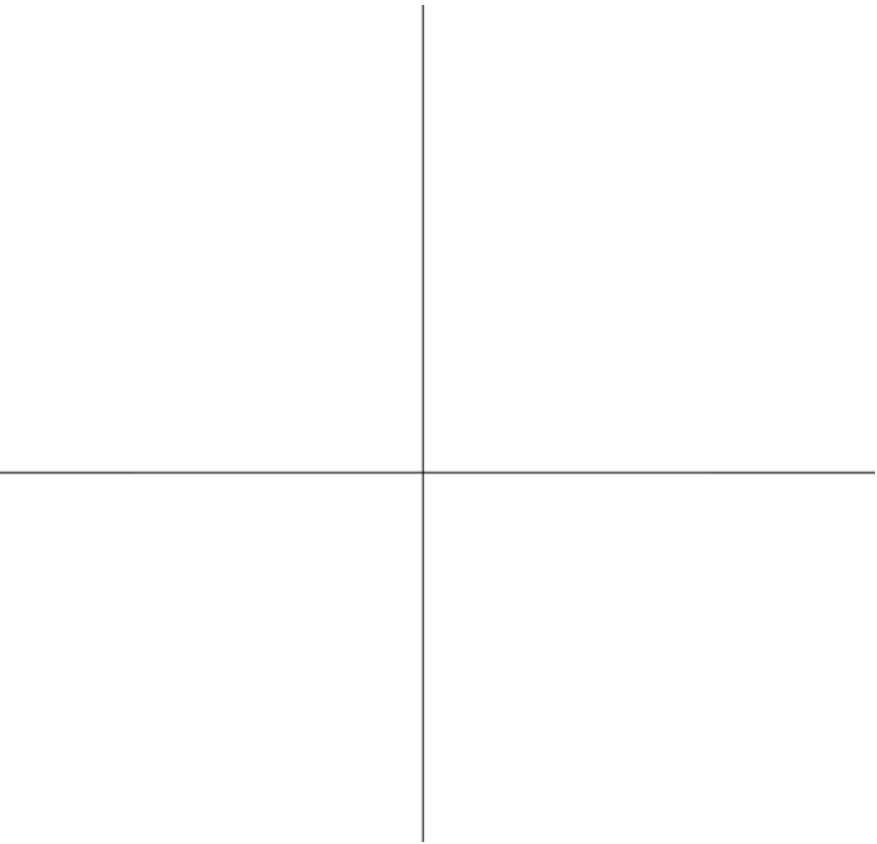
$f(x) = e^x$ Domain: $-\infty < x < \infty$;
Range: $0 < y < \infty$.



Key Values

- $\ln(1) =$
- $\ln(e) =$
- What does your calculator say about $\ln(0)$?

$f^{-1}(x) = \ln(x)$ Domain: $0 < x < \infty$;
Range: $-\infty < y < \infty$.



Key Facts and Solving:

- $\ln(a^b) = b \ln(a)$ (most important for solving)
- $\ln(ab) = \ln(a) + \ln(b)$
 - We only need $\ln(x)$ because $\log_b(y) = \frac{\ln(y)}{\ln(b)}$

Example 1: Solve the equations $e^x = 4$ and $10^x = 4$.

Example 2: Solve $2 \cdot 6^x - 1 = 12$

Example 3: (HW Q3) Solve $\log_3(\ln(x)) = 2$

Example 4: (HW Q2)

Rewrite each function in the form $y = A_0 e^{at}$

(c) $y = -2(1.599)^t - 2$

Example 5: (Combine with other skills)

Solve $3\sqrt{1 + e^{(\pi^x)}} = \log_7(7^{12})$

Example 6: (A page from Ostroff Exam 2 Spring 2014)

Solve

(a) $5000 = 2000x^9$

(b) $e^{5x} = 12$

(c) $\ln(3x + 2) = 4$

(d) $x + 3 = \frac{15}{x} + 1$

Old exam problem (Naehrig W24 Exam 2)

The population growth of City A and B are exponential.

Assume $t = 0$ corresponds to the year 2000.

(a) City A had a population of 24,000 people in 2000 with a doubling time of 50 years. When was City A's population at 17,000 people?

(c) In what year will the cities have the same population?

(b) City B had a population of 50,000 people in 2000 and its population grows by 5% in 5 years. What is the doubling time of City B's population?

Please dig into the chapter 12 homework! You have the tools you need, but please start and ask me questions.