Ch 10/11: Exponential Functions and Models

Goal: Graph and find exponential functions.

Key Ideas in Chapter 10 (Exponential Model)

$y = ab^x$, where a = "*y-intercept*" and b = "*base*" (or "*multiplier*")

$b > 1$ gives exponential growth

$0 < b < 1$ gives exponential decay

Entry Task:

a. Compute the following (no calculator):

$2^2 2^3 =$

$(4^{1/2})^3 =$

$9^{-1/2} =$

$\frac{20}{5} \cdot \frac{15}{2^3} \cdot \frac{1}{30} =$

b. Simplify/Rewrite

$x^2 x^3 =$

$(3^x)^4 =$

$4^{-x/2} =$

$\frac{6}{2^{-3x-1}} =$

Note any "rules" you observe…

Can you do this homework now?

Problem 10.2. Put each equation in standard exponential form:

(a) $y = 3(2^{-x})$

(b) $y = 4^{-x/2}$

(c) $y = \pi^{\pi x}$

(d) $y = 1\left(\frac{1}{3}\right)^{3+\frac{x}{2}}$

(e) $y = \frac{5}{0.345^{2x-7}}$

(f) $y = 4(0.0003467)^{-0.4x+2}$

"standard form" means $y = ab^x$

Exponential Functions and Graphs

Example 1: Let $y = f(x) = 5 \cdot 3^{2x}$

(a) What is *a* and what is *b*?
(b) Sketch a graph.

Example 2: Let $y = g(x) = 6 \cdot 2^{-x-1}$

(a) What is *a* and what is *b*?
(b) Sketch a graph.

Finding the model

Example 1: (*from Fall 2017 Final Exam)*

- In 2008 the population of a town was 1200.
- In 2012 the population was 1414.

Give an exponential model for the population, $f(t) = ab^t$, where t is years since 2008 (*i.e.* take $t = 0$ in 2008).

Example 2: (*from Winter 2018 Ostroff Midterm* 2)

You placed some plums in an icebox.

- 10 hours from now, the plums will be 11° Celsius.
- 22 hours from now, the plums will be 5° Celsius.

Give an exponential model for the temperature, $p(t) = ab^t$, of the plumbs t hours from ***now***.

Doubling, Tripling, & Percentage change

Key fact: Percentage change is unaffected by the initial value.

Example 1:

- The population of Rabbitown doubles every 3 months.
- The initial population (now) is 50 Rabbits

Give an exponential model for the population, $R(t) = ab^t$

Example 2:

- The population of Squirrelville triples every 10 months.
- The initial population is unknown.

Find the base in the exponential, $S(t) = ab^t$

Changes as percentages

Example 3:

Your lab has 10 grams of plutonium that has been sealed and stored away since 1984.

In 2004, it was measured that 80% of the sample remained (i.e. it had decayed by 20%).

Give an exponential model for the mass of plutonium in terms since 1984, $m(t) = ab^t$

Follow-up:

How many grams of plutonium will there be in 2034?

Next time: Chapter 11…

We will talk about the special base

$$e \approx 2.71828182\ldots$$

For now,

- sketch a graph of $y = e^x$ and
- sketch a graph of $y = e^{-x}$.

Both of those are example of exponential functions and you have the tools to sketch a rough graph now. Those two graphs are important to know as you go into Math 124.