Math 120 Chapter 7 through 12 Review

This review is not all inclusive. You are expected to know how to do all the problems in the homework.

1. **Chapters 7 - Application of Quadratic Modeling** - Know how to set up problems involving quadratic modeling and how to find optimum values (max/min). Remember, the general quadratic model is: $y = ax^2 + bx + c$.

   - If $\mathbf{a > 0}$, then the parabola opens upward, so the vertex, $x = -\frac{b}{2a}$ gives the x coordinate of the **minimum** point on the graph.
   - If $\mathbf{a < 0}$, then the parabola opens downward, so the vertex, $x = -\frac{b}{2a}$ gives the x coordinate of the **maximum** point on the graph.
   - The two scenarios above are the most common encountered in realistic story problems (*i.e.* usually you are being asked to find the vertex in these problems).
     However, if $\mathbf{a > 0}$ and you are asked for the maximum or if $\mathbf{a < 0}$ and you are asked for the minimum, then you are NOT being asked to find the vertex. In these situations, the answer is at either the highest or lowest reasonable value of x (you should compare the output from the highest reasonable x and the lowest reasonable x). You were asked to do this once in the homework on the last part of 7.11 (Wire Tricks), in that problem the highest or lowest cuts corresponded to using all the wire for a circle or all the wire for a square (the circle gave the largest area).
   - You should know how to find a quadratic model given three points as you did in HW 7.12.
   - We did all the problems in four steps in lecture:
     (a) LABEL - Label all unknowns and *clearly* write in words what each label represents.
     (b) FACT - Write down the equations for all given facts (typically this gives one equation).
     (c) WHAT TO OPTIMIZE - Write down the function what you are being asked to optimize. Use the given FACT (solving for one of the variables), to substitute into the function you want to optimize so that you get a one variable function.
     (d) OPTIMIZE/INTERPRET - The function you get should involve a quadratic function. Use facts about a parabola to find the x coordinate of the optimal value of interest (typically the vertex). Then plug this x in elsewhere to answer the original question.
   - Here are the types of problems we did in class and in the homework:
     – **Orchards/Ticket Sales (HW 7.7 and 7.8)**:
       (a) LABEL - In these scenarios, we let x='trees' and y='apples per tree' or x='price per ticket' and y='tickets sold' or x='tickets sold' and y='price per ticket' depending on the problem.
       (b) FACT - Then we given information about x and y (usually we are told that y is a linear function of x). So we find the equation for the linear relating y and x.
       (c) WHAT TO OPTIMIZE - Usually you are asked to optimize TOTAL APPLES or TOTAL MONEY, in these scenarios (using my labels above) the TOTAL $= xy$. And we substitute the function for the FACT in for y to get a TOTAL function that involves only x.
       (d) OPTIMIZE - Find the vertex and answer the question.
     – **Areas (HW 7.9, 7.10, and 7.11)**:
       (a) LABEL - Label the dimensions of the shape.
       (b) FACT - Usually information is given about the sum of the dimensions (perimeter). Write this down and solve for one of the variables. (You should know the formula for the circumference of a circle as in 7.9 and 7.11).
       (c) WHAT TO OPTIMIZE - Write down the formula for the area. Use the fact to turn the formula into a one variable function.
       (d) OPTIMIZE - Find the vertex and answer the question.
     – **Minimum distance and uniform linear motion (HW 7.13 and 7.14)**
       (a) LABEL - Make sure to clearly write two sets of equations $x = a + bt$ and $y = c + dt$ for the first person and $x = a + bt$ and $y = c + dt$ for the second person (a, b, c, and d will be different for each person).
       (b) FACT - Use the given information to find a, b, c and d for each (just like we did at the end of ch. 4).
       (c) WHAT TO OPTIMIZE - Put the x and y coordinate formulas for the two people in the distance formula. This gives a one variable function of t. Simplify/expand everything under the radical.
       (d) OPTIMIZE - Find the vertex of the parabola and answer the question.

2. **Chapter 8 and 9 - Composition and Inverses**

- Given two functions, understand how to find the function $f(g(x))$, $g(f(x))$, and $f(f(x))$ and be able to go in reverse (that is, given a complicated function can you view it as a composition of two simpler functions). You computed compositions in HW 9.6 and you did it in reverse in 8.2.
- Understand how to do compositions even if there are extra unknowns in the functions and know how to find a linear function $g(x)$ such that $g(g(x))$ is some given linea function (like in 8.3).
- Understand the basics of domain and range.
- If a function $y = f(x)$ has an inverse (*i.e.* passes the horizontal line test), then we say f has an inverse and we write $x = f^{-1}(y)$. (That means there is a function for getting back to x from y).
- When asked to find an inverse, here is what you do:

  Solve in order to get x by itself in terms of y (and that is it)!

- Know the algebra for solving for x in terms of y in all of the following scenarios:
  (a) Linear functions (as in HW 9.2a, 9.7ab)
  (b) Functions involving fractions (as in HW 9.2b, 9.5)
  (c) Quadratic functions (as in HW 9.3, 9.7cd)
  (d) Functions involving roots (as in HW 9.2cde, 9.7e)
- Understand how to break up the domain of functions that are not invertible in order to get invertible functions (as you see in HW 9.3cd, 9.7d)

3. **Chapter 10, 11, 12 - Exponential Models and Logarithms**

- Know the exponential model and what the constants represent. We have two forms;

  $$y = y_0 b^x \quad \text{and} \quad y = y_0 e^{kx}.$$

  These are two ways to write exactly the same thing. The relationship between b and k is

  $$b = e^k \quad \text{and} \quad \ln(b) = k.$$

- Know your exponent and logarithm rules. Most importantly you should know:

  $$\begin{array}{ll} (x^a)^b = x^{ab} & \ln(a^b) = b\ln(a) \\ x^a x^b = x^{a+b} & \ln(ab) = \ln(a) + \ln(b) \\ \frac{x^a}{x^b} = x^{a-b} & \ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b) \end{array}$$

  The first row in this list are the rules we use most often with the exponential model and logarithms.
- Know how to find an exponential model when given information in various forms as in HW 11.1, 11.2, 12.7, 12.9, 12.10, 12.12 and 12.13. Here are some specific notes:
  - 12.4 refers to a special model at the bottom of page 151 (if this were on an exam, I would give you the form of the model)
  - 12.7 is a great review of the models we know.
  - 12.12, the growth rate is 2.5% means that the multiplier $b = 1.025$ each year.
- Understand the definition of the logarithm:

  $$y = e^x \text{ is the same as saying } \ln(y) = x.$$

  $$y = b^x \text{ is the same as saying } \log_b(y) = x.$$

- Understand how to answer questions about doubling/tripling/quadrupling. Doubling means $y = 2y_0$, tripling means $y = 3y_0$, quadrupling means $y = 4y_0$. Understand that no matter what y_0 is, the time it takes to double/triple/quadruple is unaffected (*i.e.* y_0 always cancels).
- **You only use logarithms to solve an equations if the variable you are solving for is in the exponent**. Understand how to solve with logarithms:
  (a) Isolate the exponential (by simplifying and using exponent rules)
  (b) Take the natural logarithm of both sides.

(c) Use the logarithm rules and solve for x.

- **If you are solving an equation were the variable is in the base, then you use roots to solve.** For example, $b^5 = 10$ would be solved by taking fifth roots, that is $b = 10^{1/5}$. To solve $5^x = 10$, we use logarithms to get $\ln(5^x) = \ln(10)$, so $x \ln(5) = \ln(10)$, which gives $x = \ln(5)/\ln(10)$.
- All other logarithms can be converted to the natural logarithms using the formula:

$$\log_b(x) = \frac{\ln(x)}{\ln(b)}.$$