MATH 120 — Additional Examples and Hints Ch. 12 and 13

12.10 (b) $f(x) = \frac{ax+b}{x+d}$ with $f(0)=10$, $f(5)=4$, $f(20)=3$.

PLUGGING IN INFO

① $f(0)=10 \Rightarrow 10 = \frac{a(0)+b}{(0)+d} \Rightarrow \boxed{b = 10d}$

② $f(5)=4 \Rightarrow 4 = \frac{a(5)+b}{(5)+d} \Rightarrow \boxed{20+4d = 5a+b}$

③ $f(20)=3 \Rightarrow 3 = \frac{a(20)+b}{(20)+d} \Rightarrow \boxed{60+3d = 20a+b}$

COMBINING AND SOLVING

Using ① in ② and ③ gives

② $20+4d = 5a+10d \Rightarrow \boxed{20 = 5a+6d}$

③ $60+3d = 20a+10d \Rightarrow \boxed{60 = 20a+7d}$

Solving for a in ② gives $5a = 20-6d \Rightarrow \boxed{a = \frac{20-6d}{5}}$

Using this in ③ gives

$$60 = 20\left(\frac{20-6d}{5}\right) + 7d$$
$$60 = 4(20-6d) + 7d$$
$$60 = 80 - 24d + 7d$$
$$-20 = -17d \Rightarrow d = \frac{20}{17} \approx 1.176470588\overline{24}$$

PLUGGING THIS VALUE IN ABOVE GIVES

$$a = \frac{20-6d}{5} = 4 - \frac{6}{5}\left(\frac{20}{17}\right) = \frac{44}{17} \approx 2.588235294\overline{11}$$
$$b = 10d = 10\left(\frac{20}{17}\right) = \frac{200}{17} \approx 11.764705882\overline{4}$$

Thus, $f(x) = \dfrac{\frac{44}{17}x + \frac{200}{17}}{x + \frac{20}{17}} = \dfrac{44x+200}{17x+20}$

$= \dfrac{2.588235x + 11.76470588}{x + 1.176470588}$

ALL CORRECT !

Checking work: $f(0) = \frac{200}{20} = 10$ ✓

$f(5) = \frac{44(5)+200}{17(5)+20} = \frac{420}{105} = 4$ ✓

$f(20) = \frac{44(20)+200}{17(20)+20} = \frac{1080}{360} = 3$ ✓

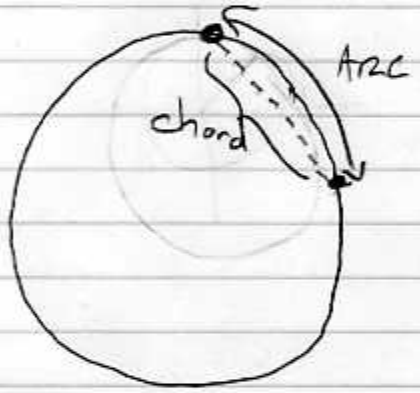
Ch. 13 Hints

On 13.3, the area of an object of the form

is equal to the area of the large wedge

minus the area of the smaller wedge

On 13.5,
if θ is small,
the length of the chord $\simeq$ length of arc
So they are asking you
to compute the arc length
with the given radius and angle.



On 13.8, set up an equation
involving arc length and an equation
involving area of a wedge
Combine the equations and solve.