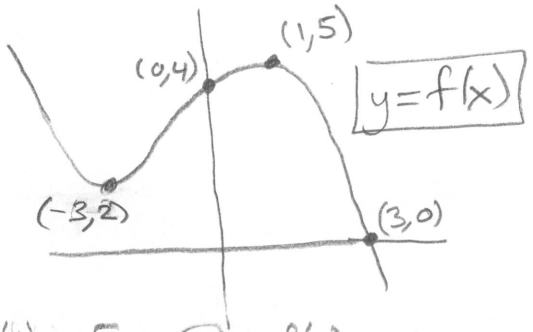
SHIFTING / DILATING / REFLECTING ANOTHER PERSPECTIVE

Consider the graph:



From the given points, we have the relationships

① $f(-3)=2$, ② $f(0)=4$, ③ $f(1)=5$, ④ $f(3)=0$.

Now assume we want to graph $\boxed{y=2f(3x-1)+4}$

Moving all the "outside" changes to the y-side gives ↓

$y-4=2f(3x-1)$, so $\boxed{\frac{1}{2}(y-4)=f(3x-1)}$

Thus, $y=f(x)$ has changed to $\frac{1}{2}(y-4)=f(3x-1)$. So "x" → "$3x-1$" and "y" → "$\frac{1}{2}(y-4)$".

NEW PERSPECTIVE — We can simply write the facts from the points above and translate over.

OLD: $y=f(x)$ NEW: $\frac{1}{2}(y-4)=f(3x-1)$

① $2=f(-3)$ so when ⓘ $\frac{1}{2}(y-4)=2$ we must have ⓘⓘ $3x-1=-3$

Solving gives:

ⓘ $\frac{1}{2}(y-4)=2$ ↓ mult. by 2; $y-4=4$ ↓ add 4; $y=8$

ⓘⓘ $3x-1=-3$ ↓ add 1; $3x=-2$ ↓ divide by 3; $x=-\frac{2}{3}$

Thus, $(-3,2)$ moves to this, $\boxed{(-\frac{2}{3},8)}$

Now do the same thing with the other points.

(2)

$$\underbrace{4}_{\frac{1}{2}(y-4)} = f(\underbrace{0}_{3x-1})$$

Thus, $\frac{1}{2}(y-4)=4 \xrightarrow{\cdot 2} y-4=8 \xrightarrow{+4} y=12$ when

$3x-1=0 \xrightarrow{+1} 3x=1 \xrightarrow{\div 3} x=\frac{1}{3}$

$(0,4)$ becomes $\boxed{(\frac{1}{3},12)}$

(3)

$$\underbrace{5}_{\frac{1}{2}(y-4)} = f(\underbrace{1}_{3x-1})$$

Thus, $\frac{1}{2}(y-4)=5 \xrightarrow{\cdot 2} y-4=10 \xrightarrow{+4} y=14$ when

$3x-1=1 \xrightarrow{+1} 3x=2 \xrightarrow{\div 3} x=\frac{2}{3}$

$(1,5)$ becomes $\boxed{(\frac{2}{3},14)}$

(4)

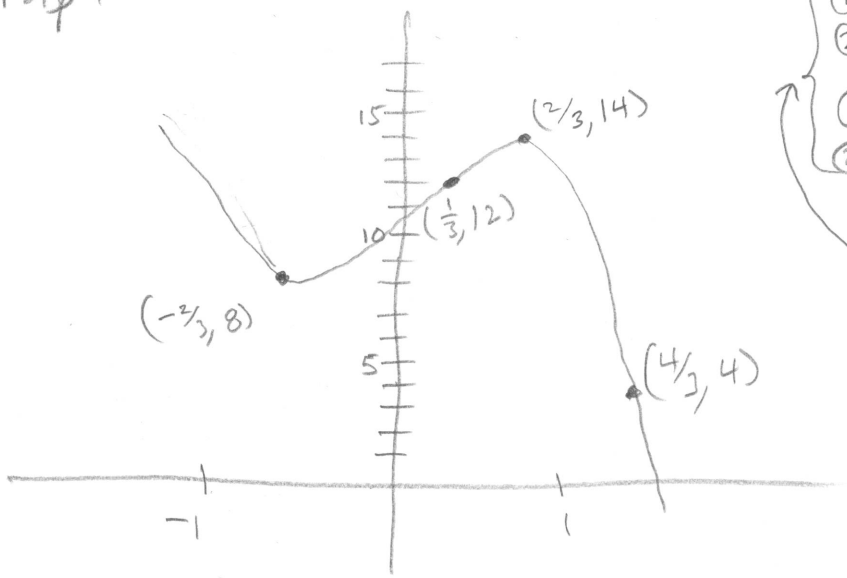
$$\underbrace{0}_{\frac{1}{2}(y-4)} = f(\underbrace{3}_{3x-1})$$

Thus, $\frac{1}{2}(y-4)=0 \xrightarrow{\cdot 2} y-4=0 \xrightarrow{+4} y=4$ when

$3x-1=3 \xrightarrow{+1} 3x=4 \xrightarrow{\div 3} x=\frac{4}{3}$

$(3,0)$ becomes $\boxed{(\frac{4}{3},4)}$

New graph:



NOTE: IN EACH CASE WE

① ADD 1 to all x-coord.
② DIVIDE ALL x-coord by 3.

① MULT ALL y-coord by 2.
② ADD 4 to ALL y-coord.

which is what I SUMMARIZED in class. This is just another perspective.