Math 120 Exam 1 Review

This review is not all inclusive. You are expected to know how to do all the problems in the homework.

1. **Chapters 1 and 2 - Unit Conversion/Rates and Coordinate Systems -** Know how to work with rates via either unit conversion or using the rate formula and be comfortable with the distance formula.

   - DISTANCE = SPEED×TIME, SPEED = $\frac{\text{DIST}}{\text{TIME}}$, TIME = $\frac{\text{DIST}}{\text{SPEED}}$ (In general, RATE = $\frac{\Delta\text{QUANTITY}}{\Delta\text{TIME}}$)
   - Know how to convert between inches/yards/feet/miles, sec/min/hours, and m/km.
   - Know how to use the distance formula,

   $$\text{DISTANCE BETWEEN TWO POINTS ON A STRAIGHT LINE } = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

   - If a question asks "How long..." or "When..", you should recognize that you first need to find the two locations the problem is referring to, then figure out the distance between the two locations, and find the time by using distance divided by speed.
   - Be comfortable with introducing variable/using speed and describing locations like you did in the Bungie problem, the running toward a puddle problem, the Ferrari/Mercedes problem, the Steve/Elsie problem, the location of the Ferry boat, and the silo problems.
   - Use a systematic approach to the problems (I have suggested the V.E.T.S. method, my own personal made-up acronym):

     (a) **Visualize**: Choose an origin, label all known points, label unknown points and quantities with variables.

     (b) **Equations**: Introduce more variables if there are any other quantities of interest and clearly write what each variable represents. Then find equations for all curves that we know equations for (we currently know the equations for circles, semicircles, lines (path), and lines with time(uniform linear motion)).

     (c) **Translate**: Translate the question into mathematical terms involving your variables and equations (at this point you may have to go back and introduce more variables or find more equations). Often we are trying to find intersections or distances.

     (d) **Solve**: Solve for what you can. Even if you don't know how you are going to finish the problem, plug in the facts that you know and solve for all intersection points. After you solve, go back and translate what you have and perhaps solve some more.
     At the very end of a problem, STOP before moving on. Go back and reread the problem and make sure that you have given the answer that the question was asking for.

2. **Chapter 3 and 4: Circles, Lines, and Uniform Linear Motion Models -** Know the equations for circles, lines and uniform linear motion.

   - The equation for a circle is $(x - h)^2 + (y - k)^2 = r^2$ where (h, k) is the center and r is the radius.
   - The equation for the upper and lower semicircles (resp.) are $y = k+\sqrt{r^2 - (x - h)^2}$ and $y = k-\sqrt{r^2 - (x - h)^2}$.
   - The equation for any horizontal line is of the form $y = k$ and the equation for any vertical line is of the form $x = h$.
   - All equation for any non-vertical line can be written in the form: $y = m(x - x_1) + y_1$ where $m = \frac{y_2 - y_1}{x_2 - x_1}$ and (x_1, y_1), (x_2, y_2) are **any** two points on the line.
   - When asked for *the equations of linear motion*, or *the parametric equations of motion*, or *the equations of uniform linear motion*, you use the following equations:

   $$x = a + bt \quad , \quad y = c + dt.$$

   - Typical Problems:

     (a) **Be able to find the intersection of a line and circle**. Often you have to use the quadratic formula which says that the solutions to $ax^2 + bx + c = 0$ are of the form $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. These problems take some time, but you have done them so often in your homework and test prep, that you should be much faster at them by the time you take the exam. (This is like the Erik's sailboat problem and the golfing problem)

(b) **Be able to find a linear models for data** (such as the Seattle and Port Townsend average home sale price problems). And be able to compare the models (when they are the same, when one is twice the other, *etc.*)

(c) **Finding the location on a linear path that is closest to a given point.** This is a procedure you used several time (on the golf problem, the mary/tiff/angela problem, and on the rome/paris/florence problem).

i. Find the equation for the linear path.

ii. Find the equation of the perpendicular line through the given point (that is, the point that you want to be close to). If m_1 is the slope of the linear path, then the slope of the perpendicular line is $m_2 = -\frac{1}{m_1}$.

iii. Find the intersection of these lines.

iv. Use this point to answer the particular question.

(d) **Find the equations for uniform linear motions**. You did this in the Margot problem and in the Juliet and Mercutio problem.

i. First, write down $x = a + bt$ and $y = c + dt$.

ii. From the given information label two points and the corresponding time when the object is at the two points (sometimes you have to use time=dist/speed to find one of the times).

iii. Plug in the information from the last step into the linear motion equations (you will get four equations). And solve for a, b, c, and d.

iv. Use the model to answer what the question is asking.

3. **Chapters 5 and 6: Functions/Graphs and multipart functions** - Know how to use functional notation, be able to read graphs, and be able to use multipart function (mechanically and in applications)

- Given a function $f(x)$, you should be able to compute and simplify $f(BLAH)$ for any 'BLAH'. You should also be able to compute things of the form $(f(x+h) - f(x))/h$ or $f(2x) - f(3x) + f(x)$, or anything that involves functional notation. You need to be able to read functional notation before the first exam.
- Know what a function is and understand the terms domain and range.
- Understand the terms positive/negative/zeros and increasing/decreasing pertaining to a function or graph.
- Be able to look at and use a multipart function (as you have to do in 6.1, 6.2, 6.3, and 6.4) You should know the multipart rule for $|x|$.
- Be able to find a formula as you do in 6.5, 6.6, 6.9, and 6.12 for a situation or graph. For 6.5 and 6.6 recognize that your first step on such a problem should be to introduce a coordinate system at the lower left corner of the object and label all points. Then the height at x is the same as y, so to find a formula for y all you have to do is find the equations for the lines.

4. **Chapter 7: Quadratic Modeling** - Understand what a function of the form $f(x) = ax^2 + bx + c$ looks like.

- Be able to find the vertex (the x coordinate of the vertex is given by $x = -\frac{b}{2a}$). And understand that if $a > 0$, then the parabola opens upward ('smiles') and if $a < 0$, then the parabola opens downward ('frowns'). From this you should be able to get a quick sketch of any parabola you are given.
- Be able to find a quadratic model when given three points by plugging in the three points to find three equations and then combine these equations to get a, b, and c.
- Be able to use parabolas on certain maximum/minimum problems (such as in 7.5) by finding a function for the quantity you are trying to maximize/minimize and then using the vertex formula.