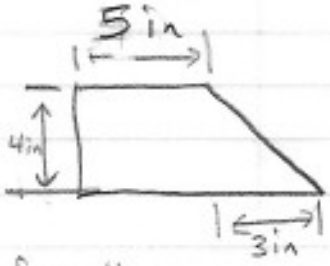
MULTIPART RULE APPLICATION PROBLEMS

Here are a couple of applied multipart function problems

EXAMPLE 1

① A cake has the following shape:

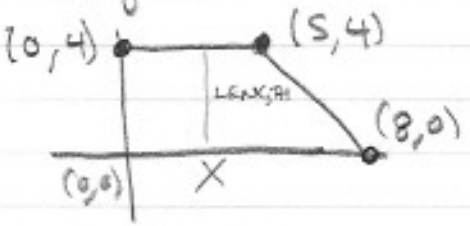


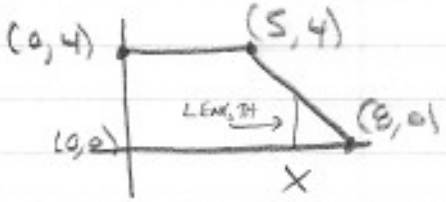
A vertical cut is made through the cake at a location x inches from the left.

ⓐ Depending on x give a multipart rule for the length of the cut.

Solution

ⓘ Impose a coordinate system where the lower left corner is the origin.





ⓘⓘ Find equations for the lines

HORIZONTAL LINE $y = 4$

OTHER LINE FROM $(5,4)$ to $(8,0)$

$y = m(x - x_1) + y_1$ $\quad m = \frac{0-4}{8-5} = -\frac{4}{3}$

$y = -\frac{4}{3}(x-8) + 0$

ⓘⓘⓘ When do you use each equation?

If $0 \le x \le 5$, then the LENGTH $= 4$ (no matter where the cut is made)

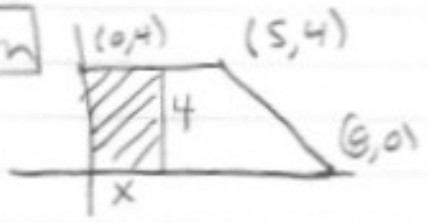
If $5 \le x \le 8$, then the LENGTH $= -\frac{4}{3}(x-8)$

(iv) Thus, LENGTH OF CUT $= \begin{cases} 4 & , \text{if } 0 \le x \le 5; \\ -\frac{4}{3}(x-8) & , \text{if } 5 \le x \le 8. \end{cases}$

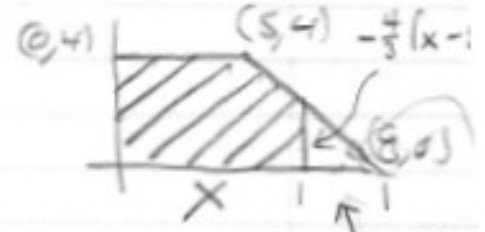
(b) Give a multipart rule for the area of the slice to the left of the cut at x.

Solution

(i)



If $0 \le x \le 5$,
then REGION = RECTANGLE
So AREA = LENGTH · WIDTH
$= 4x$



If $5 \le x \le 8$
then
AREA = AREA OF WHOLE CAKE − AREA OF SMALL TRIANGLE

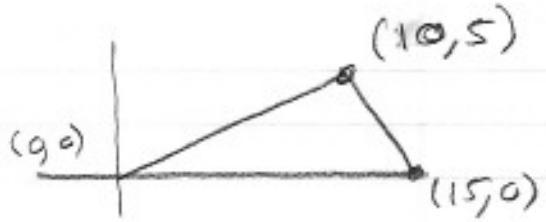
$$\text{AREA} = \begin{cases} 4x & , \text{if } 0 \le x \le 5; \\ 26 - \frac{1}{2}(8-x)\left(-\frac{4}{3}(8-x)\right), & \text{if } 5 \le x \le 8. \end{cases}$$

WHOLE AREA $= \underbrace{4 \cdot 5}_{\text{rectangle}} + \underbrace{\frac{1}{2}(3)(4)}_{\text{triangle}}$
$= 20 + 6 = 26$

$$\text{AREA} = 26 - \frac{1}{2}(8-x)\left(-\frac{4}{3}(x-8)\right)$$
(BASE, HEIGHT)

EXAMPLE 2

② Consider the graph:



ⓐ Depending on x, give a multipart rule for y.

SOLUTION

(i) Visualize/Impose Coord ✓ (DONE)

(ii) Equations

LINE 1: $y = m(x - x_1) + y_1$, $m = \frac{5-0}{10-0} = \frac{1}{2}$

$y = \frac{1}{2}(x-0) + 0$

$\boxed{y = \frac{1}{2}x}$

LINE 2: $y = m(x - x_1) + y_1$, $m = \frac{0-5}{15-10} = -1$

$y = -1(x-15) + 0$

$\boxed{y = -x + 15}$

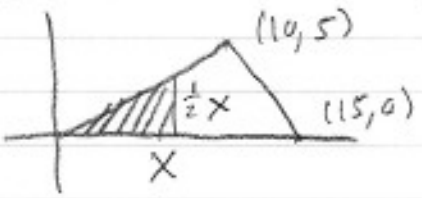
(iii) When do you use each?

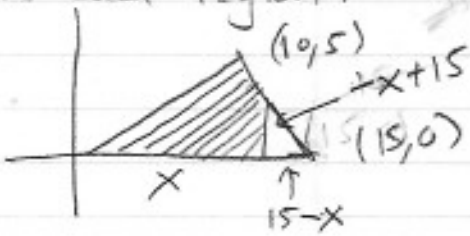
$$\boxed{y = \begin{cases} \frac{1}{2}x, & \text{if } 0 \le x \le 10; \\ -x+15, & \text{if } 10 \le x \le 15. \end{cases}}$$

ⓑ Depending on x, give the area to the left of a vertical line through the triangle at x.

SOLUTION

(i) visualize the shape created in each region.





AREA = $\frac{1}{2}$(BASE)(HEIGHT)

$= \frac{1}{2} \cdot x \cdot \frac{1}{2}x$

$= \frac{1}{4}x^2$

AREA = WHOLE − SMALL TRIANGLE

$\frac{1}{2}$(BASE)(HEIGHT) − $\frac{1}{2}$(BASE)(HEIGHT)

$\frac{1}{2} 15 \cdot 5 - \frac{1}{2}(15-x)(-x+15)$

$\frac{75}{2} - \frac{1}{2}(15-x)^2$

↑ same as $15-x$

$$\boxed{\text{AREA} = \begin{cases} \frac{1}{4}x^2, & \text{if } 0 \le x \le 10; \\ \frac{75}{2} - \frac{1}{2}(15-x)^2, & \text{if } 10 \le x \le 15. \end{cases}}$$