1. (11 pts) The number of grey hairs on a particular instructor's head are growing exponentially. When the instructor turns 25 years old, he has 100 grey hairs. When the instructor is turns 30 years old, he has 500 grey hairs.

(a) How many grey hairs will the instructor have when he turns 37 years old?

Let x = years beyond 25 (so $x=0 \Leftrightarrow$ 25 years old).

y = # of gray hairs

$y = y_0 b^x$

① $x=0,\ y=100 \Rightarrow 100 = y_0 b^0 \Rightarrow y_0 = 100$

② $x=5,\ y=500 \Rightarrow 500 = 100 b^5 \Rightarrow 5 = b^5 \Rightarrow b = 5^{1/5} \approx 1.379729661$

$$\boxed{y = 100\,(5^{1/5})^x = 100\,5^{x/5} = 100\,(1.379729661)^x}$$

$37 \Leftrightarrow x = 12$

$y = 100\,5^{12/5} = 100 \cdot 5^{2.4} = 4759.134847$ grey hairs

$\approx \boxed{4759 \text{ grey hairs}}$

(b) How long does it take for the number of grey hairs to triple?

$300 = 100\,5^{x/5}$

$3 = 5^{x/5}$

$\ln(3) = \frac{x}{5}\ln(5)$

$x = \dfrac{5\ln(3)}{\ln(5)} \quad \left(= \dfrac{\ln(3)}{\ln(1.379729661)} \right)$

$\boxed{x \approx 3.413030972 \text{ years}}$

2. (12 pts) Molly and Phil are born on the same day. Both of them get standardized intelligence test scores each year as they grow older. Phil's scores are modeled by the linear-to-linear rational model:

$$P(x) = \frac{180x}{x+1},$$

where x is his age, and $P(x)$ are the points he gets on the test.

Molly's scores are also given by a **linear-to-linear rational function**.

When Molly is 4 years old, she scores 100 points on the test.
When Molly is 10 years old, she scores 150 points on the test.
As Molly gets older and older her scores on the test approach 200.

(a) Find Molly's linear-to-linear function, $M(x)$, for her test scores in terms of her age, x.

$$M(x) = \frac{ax+b}{x+d}$$

① Horiz. Asymptote at $y = 200 \Rightarrow a = 200$

② $M(4) = 100 \Rightarrow 100 = \frac{200(4)+b}{(4)+d} \Rightarrow 400 + 100d = 800 + b$

$100d - 400 = b$

③ $M(10) = 150 \Rightarrow 150 = \frac{200(10)+b}{(10)+d} \Rightarrow 1500 + 150d = 2000 + b$

② & ③ $\Rightarrow 1500 + 150d = 2000 + 100d - 400$

$50d = 100$

$d = 2$

$b = 100d - 400 = -200$

$$\boxed{M(x) = \frac{200x - 200}{x+2}}$$

check:

$M(4) = \frac{200(4)-200}{4+2} = \frac{600}{6} = 100$ ✓

$M(10) = \frac{200(10)-200}{(10)+2} = \frac{1800}{12} = 150$ ✓

(b) At what age do Phil and Molly have the same test score?

$$\frac{180x}{x+1} = \frac{200x-200}{x+2}$$

$180x(x+2) = 200(x-1)(x+1)$

$180x^2 + 360x = 200x^2 - 200$

$0 = 20x^2 - 360x - 200$

$0 = x^2 - 18x - 10$

$$x = \frac{18 \pm \sqrt{18^2 - 4(1)(-10)}}{2} = \frac{18 \pm \sqrt{364}}{2}$$

$\frac{18 + \sqrt{364}}{2}$

≈ 18.55939

$\approx \boxed{18.56 \text{ years old}}$

3. (12 points) Dr. Loveless is walking in the coordinate plane on a straight line at a constant speed (*i.e.* **uniform linear motion**). You and some classmates are standing at the **origin** and you throw a water balloon toward Dr. Loveless. The balloon also exhibits **uniform linear motion**. Let t be the time in minutes since you threw the balloon.
At $t = 2$ minutes later, the balloon is at the location [redacted]. (6,4)

Also, you know that Dr. Loveless' location is given by the linear parametric equations: in feet

$$x(t) = \text{[redacted]}4t$$
$$y(t) = 9 - 2t.$$

(a) Find the time when the distance between Dr. Loveless and the balloon are **minimum**. (Hint: First find the linear parametric equations for the location of the balloon).

BALLOON: $x(t) = at + b$
$y(t) = ct + d$

$t = 0 \quad (x,y) = (0,0)$
$b = 0$ and $d = 0$

$t = 2 \quad (x,y) = (6,4)$
$6 = a(2) \quad a = 3$
$4 = c(2) \quad c = 2$

$$\boxed{\begin{aligned} x(t) &= 3t \\ y(t) &= 2t \end{aligned}}$$

$$\text{DISTANCE} = \sqrt{(4t - 3t)^2 + (9 - 2t - 2t)^2}$$
$$= \sqrt{t^2 + (9-4t)^2}$$
$$= \sqrt{t^2 + 81 - 72t + 16t^2}$$
$$= \sqrt{17t^2 - 72t + 81}$$

minimized when $t = -\frac{-72}{2(17)} = \frac{36}{17}$ minutes
$= 2.117647059$
$\approx \boxed{2.12 \text{ minutes}}$

(b) Between the times $t = 0$ and $t = 5$, what is the **maximum** distance between Dr. Loveless and the water balloon?

HAS TO BE AT $t = 0$ or $t = 5$

At $t = 0$, DIST $= \sqrt{17(0)^2 - 72(0) + 81} = 9$ ft

At $t = 5$, DIST $= \sqrt{17(5)^2 - 72(5) + 81} = \sqrt{146} \approx \boxed{12.083045 \text{ ft}}$

4. (a) (5 pts) Let $f(x) = \frac{3}{x}$, $g(x) = \ln(x)$, and $h(x) = 1 + (x-2)^2$.
Assuming $x \leq 2$, find the appropriate inverse function of $y = f(g(h(x)))$.

$$y = \frac{3}{\ln(1+(x-2))^2} \Rightarrow \ln(1+(x-2)^2) = 3/y$$

$$1+(x-2)^2 = e^{3/y}$$

$$(x-2)^2 = e^{3/y} - 1$$

$$x - 2 = \pm\sqrt{e^{3/y} - 1}$$

$$x = 2 \pm \sqrt{e^{3/y} - 1}$$

$$\boxed{x = 2 - \sqrt{e^{3/y} - 1}}$$

(b) (5 pts) The vertex for the absolute value function $y = |x|$ is at the point (0,0).
Find the (x, y) coordinate of the vertex of $y = 5\,|3x + 10| - 2$.
(That is, where does $(0, 0)$ get moved too?).

$$\underbrace{\tfrac{1}{5}(y+2)}_{0} = \underbrace{|3x+10|}_{0} \Rightarrow \tfrac{1}{5}(y+2) = 0 \quad \text{AND} \quad 3x+10=0$$

$$y+2=0 \qquad 3x = -10$$

$$y = -2 \qquad x = -10/3$$

$$\boxed{(-10/3, -2)}$$

(c) (5 pts) Robb is trying to decide between two pieces of pie, an apple slice or a pumpkin slice. Each slice is cut into a circular wedge (of the same thickness), but they have different radii and different angles. Robb has no preference for taste, he just wants the biggest slice, so he get's out his measuring tape and measures the radius and arc length for each piece and finds:
PIE PIECE 1 (APPLE): Radius = 4 inches, Arc Length = 3 inches.
PIE PIECE 2 (PUMPKIN): Radius = 5 inches, Arc Length = 2.5 inches.
Find the area of each circular wedge and determine which is bigger.

1. $s = \theta r$, Area $= \frac{1}{2}\theta r^2$
$3 = \theta \cdot 4$
$\theta = 3/4$ radians, Area $= \frac{1}{2} \cdot \frac{3}{4} \cdot 4^2 = 6 \text{ in}^2$

2. $2.5 = \theta \cdot 5$
$\theta = 1/2$ radians, Area $= \frac{1}{2} \cdot \frac{1}{2} 5^2 = \frac{25}{4} = 6.25 \text{ in}^2$

PUMPKIN PIE BIGGER