1. (13 points) Phineas starts walking at a constant rate on a straight line in the coordinate plane (*i.e.* he exhibits **uniform linear motion**). At time $t = 0$ seconds, Phineas is at the point $(-1, -3)$. At time $t = 10$ seconds, he is at the point $(4, 7)$.

(a) (5 pts) Find the **parametric equations** for the x and y coordinates of Phineas' location at time t.

$x = a + bt, \quad y = c + dt$

$t=0, x=-1 \Rightarrow -1 = a + b(0) \Rightarrow a = -1$

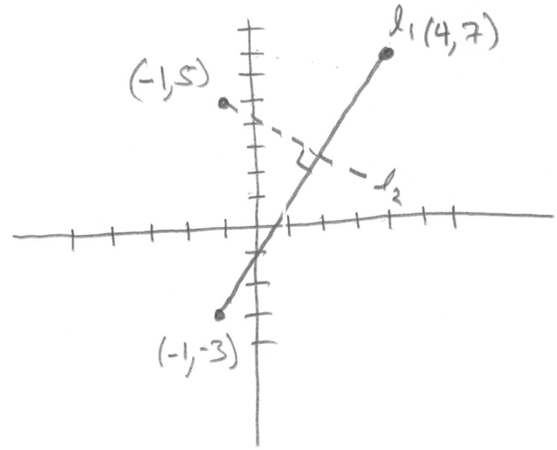
$t=10, x=4 \Rightarrow 4 = -1 + b(10) \Rightarrow b = 5/10 = 1/2$

$t=0, y=-3 \Rightarrow -3 = c + d(0) \Rightarrow c = -3$

$t=10, y=7 \Rightarrow 7 = -3 + d(10) \Rightarrow d = 1$

$$\boxed{\begin{array}{l} x = -1 + \frac{1}{2}t \\ y = -3 + t \end{array}}$$

(b) (8 pts) Ferb is standing still in the coordinate plane at the point $(-1, 5)$. At what time, t, is Phineas closest to Ferb?



ℓ_1: $m = \frac{7 - -3}{4 - -1} = \frac{10}{5} = 2$

$y = 2(x - -1) + -3$

$y = 2x + 2 - 3$

$y = 2x - 1$

ℓ_2: $m = -\frac{1}{2}$, $y = -\frac{1}{2}(x - -1) + 5$

$y = -\frac{1}{2}x - \frac{1}{2} + 5 = -\frac{1}{2}x + \frac{9}{2} = -0.5x + 4.5$

INTERSECT: $2x - 1 = -\frac{1}{2}x + \frac{9}{2}$

$\frac{5}{2}x = \frac{11}{2}$

$x = \frac{11}{5} = 2.2$

NOTE: $y = 2x - 1 = \frac{22}{5} - 1 = \frac{17}{5} = 3.4$

TIME: $x = \frac{11}{5} \Rightarrow \frac{11}{5} = -1 + \frac{1}{2}t$

$\frac{16}{5} = \frac{1}{2}t$

$\boxed{t = \frac{32}{5} = 6.4 \text{ seconds}}$

2. (12 points) For **both parts** below, consider the quadratic function $f(x) = 3x^2 - x - 4$.

(a) (6 pts) Find and simplify $xf(x+5) - xf(x)$.

$$x\left[3(x+5)^2 - (x+5) - 4\right] - x\left[3x^2 - x - 4\right]$$

$$= x\left[3(x^2+10x+25) - x - 5 - 4\right] - 3x^3 + x^2 + 4x$$

$$= \cancel{3x^3} + 30x^2 + 75x - x^2 - 5x - \cancel{4x} - \cancel{3x^3} + x^2 + \cancel{4x}$$

$$= \boxed{30x^2 + 70x}$$

(b) (6 pts) Let $g(x) = ax^2 + bx + c$ be another quadratic function for some constants a, b and c (which you will find). In addition, you know that

1. the y-intercept of $g(x)$ is 7 units above the y-intercept for $f(x)$.
2. the graphs of $g(x)$ and $f(x)$ intersect when $x = 1$, and
3. the graphs of $g(x)$ and $f(x)$ also intersect when $x = -1$, and

Find the x-coordinate of the vertex for $g(x)$.

1) $g(0) = f(0) + 7 \Rightarrow a(0)^2 + b(0) + c = 3(0)^2 - (0) - 4 + 7$

$c = 3$

2) $g(1) = f(1) \Rightarrow a(1)^2 + b(1) + c = 3(1)^2 - (1) - 4$

$a + b + c = -2$

$a + b + 3 = -2$ $\Big\} \Rightarrow$ $a + b = -5$

3) $g(-1) = f(-1) \Rightarrow a(-1)^2 + b(-1) + c = 3(-1)^2 - (-1) - 4$

$a - b + 3 = 0 \Rightarrow$ $a - b = -3$

COMBINE

② + ③ $\Rightarrow$ $2a = -8 \Rightarrow$ $a = -4$

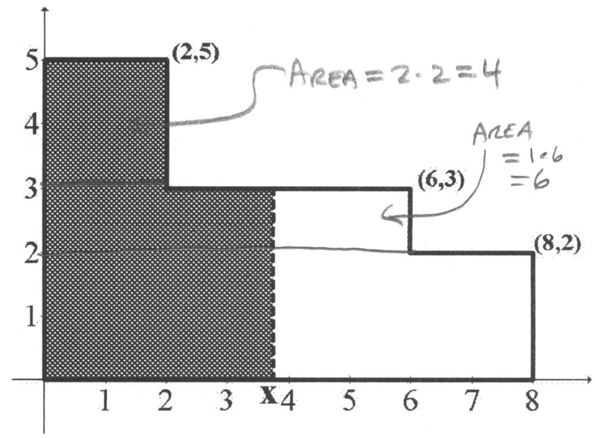
Thus, $b = -5 - b = -5 - -4 = -1$ $\quad$ $b = -1$

$$\boxed{g(x) = -4x^2 - x + 3}$$

"x-coord of vertex" $= x = -\dfrac{b}{2a} = -\dfrac{-1}{2(-4)} = \boxed{\dfrac{-1}{8}}$

3. (12 points) The floor plan for Brienne's living room is the shape given below, all units are in yards. She plans to make a vertical line separation at some location x yards from the left (x can be any value between 0 and 8). She plans to lay down carpet covering all the area to the left of the vertical lines at x. **The carpet costs \$10 per square yard.**

(a) (9 pts) Find the multipart function for the cost to carpet the area of the shaded region to the left of x.



For $0 \le x \le 2$ AREA = BASE·HEIGHT
$= x \cdot 5 = 5x$ yd^2
COST $= \frac{\$10}{yd^2} \cdot 5x$ yd^2 $= 50x$ dollars

$2 \le x \le 6$ AREA = 4 + BASE·HEIGHT
$= 4 + x \cdot 3 = 3x + 4$
COST $= 10(3x+4) = 30x + 40$

$6 \le x \le 8$ AREA = 4 + 6 + BASE·HEIGHT
$= 10 + x \cdot 2 = 2x + 10$
COST $= 10(2x+10) = 20x + 100$

$$\text{COST} = \begin{cases} 50x, & \text{if } 0 \le x \le 2; \\ 30x+40, & \text{if } 2 \le x \le 6; \\ 20x+100, & \text{if } 6 \le x \le 8. \end{cases}$$

(b) (3 pts) Brienne wants to spend exactly \$200 for this new carpet. For what value of x will the cost be exactly \$200?

NOTE: COST = \$200 $\iff$ AREA = 20 yd^2
WE CAN SEE THAT $x \ge 2$ BECAUSE $50 \cdot 2 = \$100$.
WE CAN ALSO SEE THAT $x \le 6$ BECAUSE $20 \cdot 6 + 100 = \$220$.
THUS, THE ANSWER MUST OCCUR FOR $2 \le x \le 6$.

$200 = 30x + 40 \Rightarrow 160 = 30x$
$\frac{160}{30} = x$

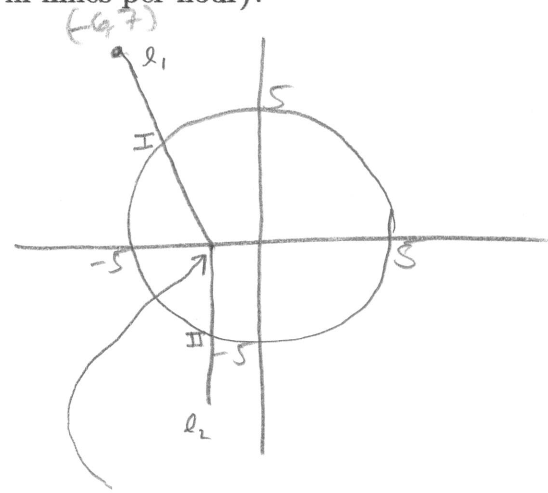
$$\boxed{x = \frac{16}{3} = 5.\overline{3} \text{ yds}}$$

4. (13 points) Samwise lives in a town with only one cell phone tower. His cell phone only works if he is within a 5 mile *circular* radius of the tower.

Samwise is sitting on a bus that is 6 miles WEST and 7 miles NORTH of the cell phone tower. The bus is traveling on a straight line toward the SOUTHERNMOST point of the circular coverage area. When the bus is directly WEST of the cell phone tower it changes direction and starts heading DUE SOUTH, eventually exiting the circular coverage area.

The bus travels at a constant speed for the entire trip and Samwise notices that he has cell phone service for exactly 20 minutes.

How fast is the bus traveling (give your final answer in miles per hour)?



CIRCLE: $x^2+y^2=5^2=25$

ℓ_1: $(-6,7)$ to $(0,-5)$

$$m=\frac{7--5}{-6-0}=\frac{12}{-6}=-2$$

$$y=-2x-5$$

TURNING POINT: $y=0 \Rightarrow$ $0=-2x-5$

$5=-2x$

$x=-5/2=-2.5$ $\quad(-2.5,0)$

INTERSECTION 1: $x^2+(-2x-5)^2=25$

$x^2+4x^2+20x+25=25$

$5x^2-20x=0$

$5x(x+4)=0$ $\quad \begin{cases} x=0 \checkmark \text{ or} \\ x=-4 \end{cases}$

$y=-2(-4)-5=3$ $\quad$ I $(-4,3)$

INTERSECTION 2: $(-2.5)^2+y^2=25$

$6.25+y^2=25$

$y^2=18.75$

$y=\pm\sqrt{18.75}\approx\pm4.330127$

II $(-2.5,-\sqrt{18.75})$

$20\text{ min}\cdot\frac{1\text{ hr}}{60\text{ min}}=\frac{1}{3}\text{ hrs}$

$$\text{SPEED}=\frac{\text{DIST}}{\text{TIME}}=\frac{\sqrt{(-4--2.5)^2+(0-3)^2}+\sqrt{18.75}\text{ mi}}{\frac{1}{3}\text{ hr}}\approx\boxed{23.0526\text{ mph}}$$

$$=\frac{\sqrt{2.25+9}+\sqrt{18.75}}{(\frac{1}{3})}=\frac{\sqrt{11.25}+\sqrt{18.75}}{\frac{1}{3}}=\frac{7.6842289\text{ mi}}{\frac{1}{3}\text{ hr}}$$