1. Bob has Sinusoidal Fever, which causes his body temperature to be a sinusoidal function of time. At 1:00 AM today, his temperature reached its lowest point, 96 degrees. His temperature will reach its highest point, 105 degrees, for the third time today at 2:00 PM.

For how much time today (from midnight to midnight) will Bob's temperature be above 100 degrees?

\[
\begin{align*}
2 \frac{1}{2} \text{ full waves in 13 hours: } & \quad \frac{13}{2.5} = 5.2 \text{ hours/wave} \\
A = \frac{105 - 96}{2} = 4.5 & \quad D = \frac{105 + 96}{2} = 100.5 \\
B = 5.2 & \quad C = 2.3 \\
\sin \left( \frac{2\pi}{5.2} (x - 2.3) \right) + 100.5 & \quad \sin^{-1}(-0.7) = \frac{2\pi}{5.2} (x - 2.3) \\
0.111341014341 = \frac{2\pi}{5.2} (x - 2.3) & \quad x = 2.2078535443 \\
\text{Symmetry Trick: } 3.6 - 2.2078535443 = 1.3921464557 & \quad 3.6 + 1.3921464557 = 4.9921464557 \\
\text{Move 100 Degrees: } & \quad 2.207 \rightarrow 4.99, 2.407 \rightarrow 10:19, \\
& \quad 12.607 \rightarrow 15.29, 17.807 \rightarrow 20:59, \\
& \quad 22.007 \rightarrow 24.
\end{align*}
\]

\[
\begin{align*}
4.9921464557 - 2.2078535443 + 24 & \quad 23.0078.035414 \\
= 12.129318127 & \quad 12.13 \text{ hours}
\end{align*}
\]
2. The town of Snub had a population of 1,000 in the year 1970. The population increases by 2.11% each year. The town of Goot had a population of 700 in the year 1977. In the year 1990, there were twice as many people in Goot as in Snub.

(a) What is Goot's doubling time?

Let $x = 0 ightarrow 1970$

\[ y = y_0 b^x \]

$x = 0, \ y = 1000 

\Rightarrow \ y_0 = 1000$

$x = 1, \ y = 1000 + 1000 \cdot 0.0211 = 1021.1 \Rightarrow \ 1021.1 = 1000b \Rightarrow \ b = \frac{1021.1}{1000} = 1.0211$

\[ y = 1000 (1.0211)^x \]

\[ x = 7, \ y = 700 \Rightarrow \ 700 = y_0 b^7 \Rightarrow \ y_0 = \frac{700}{b^7} \]

\[ x = 20, \ y = 2 \ (1000 (1.0211)^{20}) = 3036.65546 \Rightarrow \ 3036.65546 = y_0 b^{20} \]

\[ b = \frac{3036.65546}{1000} = 4.33807922557 \]

\[ y_0 = \frac{700}{(4.33807922557)^7} = 317.641953849 \]

\[ y = y_0 b^x \]

\[ 2y_0 = y_0 (1.11949686676)^x \]

\[ 2 = (1.11949686676)^x \Rightarrow \ x = \frac{\ln(2)}{\ln(1.11949686676)} \approx 6.14060163354 \text{ years after 1977} \]

(b) When will there be 10 times as many people in Goot as in Snub? Express your answer in years after 1977.

\[ \text{Goot} = 10 \times \text{Snub} \]

\[ 317.641953849 \cdot (1.11949686676)^x = 10 \times 1000 \ (1.0211)^x \]

\[ \Rightarrow \ \left( \frac{1.11949686676}{1.0211} \right)^x = \frac{10000}{317.641953849} \]

\[ \Rightarrow \ 1.09636359648 = 3.14819874357 \]

\[ \Rightarrow \ x = \frac{\ln(3.14819874357)}{\ln(1.09636359648)} \approx 37.4941029212 \text{ years after 1970} \]

\[ 30.4941029212 \text{ years after 1977} \]
3. Godzilla is attacking, but at the moment he is standing on top of a building downtown. You want to determine Godzilla's height, so you measure three angles. First, from a certain distance away from the building, you measure the angle the top of the building makes with the horizontal: $\theta_1 = 72^\circ$. You then move 50 meters farther from the building and measure the angle Godzilla's head makes with the horizontal: $\theta_2 = 74^\circ$. You then move 75 meters farther from the building and measure the angle the top of the building makes with the horizontal: $\theta_3 = 60^\circ$.

The figure may not be to scale.

\[ \begin{align*}
&\text{How tall is Godzilla?} \\
&\text{1. } \tan(72^\circ) = \frac{y}{x} \Rightarrow y = x \tan(72^\circ) \\
&\text{2. } \tan(74^\circ) = \frac{y + z}{x + 50} \Rightarrow y + z = (x + 50) \tan(74^\circ) \\
&\text{3. } \tan(60^\circ) = \frac{y}{x + 125} \Rightarrow y = (x + 125) \tan(60^\circ) \\
\end{align*} \]

Combining 1 and 3 gives \( x \tan(72^\circ) = (x + 125) \tan(60^\circ) \)

\[ x \left( \tan(72^\circ) - \tan(60^\circ) \right) = 125 \tan(60^\circ) \]

\[ x = \frac{125 \tan(60^\circ)}{\tan(72^\circ) - \tan(60^\circ)} \approx 160.8955 \]

\[ y = x \tan(72^\circ) \approx 495.1856 \]

\[ z = (x + 50) \tan(74^\circ) - y \approx 246.2946 \]
4. Maria is riding a bicycle. The rear wheel has a radius of 34.3 cm, and the front sprocket has a radius of 8.7 cm. If she travels at a speed of 27 kilometers per hour when pedaling at a rate of 96 revolutions per minute, what is the radius of the rear sprocket?

The figure may not be to scale.

![Diagram of a bicycle with labels for rear wheel, rear sprocket, and front sprocket.]

The order of my work is Labeled Below

<table>
<thead>
<tr>
<th></th>
<th>V</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>45000 cm/min</td>
<td>1311.95325277 rad/min</td>
</tr>
<tr>
<td>B</td>
<td>1670.4π cm/min</td>
<td>1311.95325277 rad/min</td>
</tr>
<tr>
<td>C</td>
<td>1670.4π cm/min</td>
<td>192π rad/min</td>
</tr>
</tbody>
</table>

\[ V_A = \frac{27 \text{ km/hr}}{1 \text{ hr}} = \frac{27 \text{ km}}{1 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{100 \text{ cm}}{1 \text{ km}} = 45000 \text{ cm/min} \]

\[ \omega_c = 96 \text{ rev/min} = \frac{96 \text{ rev}}{1 \text{ min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 192\pi \text{ rad/min} \]

We know \( \omega_A = \omega_B \) and \( V_B = V_C \)

\[ V_C = \omega_c r_c = 8.7 \times 192\pi \text{ cm/min} = \]

\[ \omega_A = \frac{V_A}{r_A} = \frac{45000}{34.3} = 1311.95325277 \frac{\text{rad}}{\text{min}} \]

\[ r_B = \frac{V_B}{\omega_B} = \frac{1670.4\pi}{1311.95325277} \approx 3.99992607203 \]

\[ 4 \text{ cm} \]
5. Alberto is planning to hike from his house in the forest directly to the village of Target, which is 10 km away from his house. Target is due north of the village of Argh. Alberto's house is 9 km due west of Argh. The village of Lump is 3 km south and 4 km west of Target.

On Alberto's hike, how close does he come to Lump?

Lump is at \( x = -4, y = \sqrt{19} - 3 \approx 1.3589939454 \)

**Equation of Alberto's path**

\[ y = m(x - x_1) + y_1 \]

\[ m = \frac{\sqrt{19} - 0}{0 - (-9)} = \frac{\sqrt{19}}{9} \approx 0.484322104878 \]

\[ y = \frac{\sqrt{19}}{9} (x + 9) \]

**Equation of perpendicular through Lump**

\[ y = -\frac{9}{\sqrt{19}} (x + 4) + \sqrt{19} - 3 \]

**Intersection**

\[ \frac{\sqrt{19}}{9} (x + 9) = -\frac{9}{\sqrt{19}} (x + 4) + \sqrt{19} - 3 \]

\[ \frac{\sqrt{19}}{9} x + \frac{9}{\sqrt{19}} = -\frac{9}{\sqrt{19}} x - \frac{36}{19} + \sqrt{19} - 7 \]

\[ \frac{28}{\sqrt{19}} x + \frac{9}{\sqrt{19}} x = \frac{-36}{19} - 7 \]

\[ x = \frac{-36}{28} \approx -4.169027476 \]

So, \( y = 2.2196952787 \)

**Distance from Lump**

\[ \sqrt{(-4.169 - 4)^2 + (2.219 - (\sqrt{19} - 3))} \approx 0.9564 \text{ km} \]
6. The figure below shows a parabolic arch. The width \( w \) between the bases of the arch is 20 feet. The height \( h \) of the arch above the ground is 33 feet. The angle \( \theta \) shown in the figure is 48°. Find the length of the line segment labeled \( k \).

\[
y = ax^2 + bx + c = a(x-h)^2 + k
\]

Given \( x = 0, y = 0 \), \( x = 20, y = 0 \) \( (10, 33) \) is the vertex

\[ x = 10, y = 33 \]

This is the longer way to find the model (this is what I would expect most students would do).

\[ 0 = a(0)^2 + b(0) + c \Rightarrow c = 0 \]

\[ 0 = a(20)^2 + b(20) \Rightarrow 0 = 400a + 20b \Rightarrow -400a = 20b \Rightarrow b = -20a \]

Combining gives:

\[ 33 = a(10)^2 + b(10) = 330a + 10b \]

Thus, \[ a = -0.33 \]

Thus, \[ y = -0.33x^2 + 6.6x \]

Also know \( \tan(48°) = \frac{k}{v} \Rightarrow k = v \tan(48°) \)

And \( (v, k) \) is a point on the parabola so \( k = -0.33v^2 + 6.6v \)

Combining gives \( v \tan(48°) = -0.33v^2 + 6.6v \)

\[ 0 = -0.33v^2 + 6.6v - v \tan(48°) = v(-0.33v + 6.6 - \tan(48°)) \]

\[ v = 0 \text{ or } -0.33v + 6.6 - \tan(48°) = 0 \Rightarrow v = \frac{6.6 - \tan(48°)}{-0.33} \approx 16.6245 \text{ ft} \]

Thus, \[ k = v \tan(48°) = \frac{(6.6 - \tan(48°)) \tan(48°)}{-0.33} \approx 18.4744922417 \text{ ft} \]
7. Find a linear function $g(x)$ such that the function $f(x) = g(g(x))$ has the properties

$$f(0) = -24 \text{ and } f(5) = -4.$$

$$g(x) = ax + b$$

$$f(x) = g(g(x)) = a(ax + b) + b = a^2x + ab + b$$

We want

1. $-24 = a^2(0) + ab + b \Rightarrow -24 = ab + b$
2. $-4 = a^2(5) + ab + b$

Combining

$$-4 = 5a^2 - 24$$

$$\Rightarrow 20 = 5a^2 \Rightarrow a^2 = 4 \Rightarrow a = \pm 2$$

$a = -2$ gives

$$-24 = -2b + b \Rightarrow -24 = -b \Rightarrow b = 24$$

$a = 2$ gives

$$-24 = 2b + b \Rightarrow -24 = 3b \Rightarrow b = -8$$

Two correct answers:

$$y = -2x + 24 \text{ or } y = 2x - 8$$
8. Let \( k(x) = \frac{2x - 8}{5x + 7} \). Find \( k^{-1}(x) \).

\[
y = \frac{2x - 8}{5x + 7} \Rightarrow y(5x + 7) = 2x - 8
\]

\[
\Rightarrow 5yx + 7y = 2x - 8
\]

\[
\Rightarrow 7y + 8 = 2x - 5yx
\]

\[
\Rightarrow 7y + 8 = (2 - 5y)x
\]

\[
\Rightarrow x = \frac{7y + 8}{2 - 5y} = k^{-1}(y)
\]

Thus, \( k^{-1}(x) = \frac{7x + 8}{2 - 5x} \).