1. (10 points) Maggie is a high jumper. The more hours that she practices, the higher her average jump height will be. If she practices for no hours, her average jump height is 3 feet. If she practices for 50 hours, her average jump height is 5 feet. As she practices more and more, her average jump height approaches (but never exceeds) 7 feet.

(a) Find the linear-to-linear rational model that gives Maggie's average jump height, $y$, in terms of the number of hours of practice, $x$.

$$y = \frac{ax + b}{x + d}$$

1. $x = 0 \Rightarrow y = 3$
2. $x = 50 \Rightarrow y = 5$
3. horiz. asymp. $y = 7$

$$y = \frac{7x + b}{x + d}$$

$$3 \Rightarrow b = 7$$

$$1 \Rightarrow 3 = \frac{7(0) + b}{0 + d} \Rightarrow b = 3d$$

$$2 \Rightarrow 5 = \frac{7(50) + b}{50 + d} \Rightarrow 250 + 5d = 350 + b$$

Combining $\Rightarrow 250 + 5d = 350 + 3d \Rightarrow 2d = 100 \Rightarrow d = 50$ and $b = 3d = 150$

$$y = \frac{7x + 150}{x + 50}$$

(b) If Maggie wants to have an average jump height of 6 feet, how many hours should she practice?

$$6 = \frac{7x + 150}{x + 50} \Rightarrow 6x + 300 = 7x + 150$$

$$x = 150 \text{ hours}$$
2. (10 points) Ron and Harry are both running counterclockwise on a circular track with radius 15 feet. Ron starts at the southernmost point and Harry is the easternmost point. Ron is running at 3 feet/sec and Harry completes one lap in 40 seconds.

(a) Give Harry’s $x$ and $y$ coordinates after 4 seconds.

$$\omega = \frac{\text{1 rev}}{40 \text{ sec}} = \frac{2\pi \text{ rad}}{40 \text{ sec}} = \frac{\pi}{20} \text{ rad/sec}$$

$$\theta_0 = 0 \text{ rad}$$

$$\theta = \omega t + \theta_0 = \frac{\pi}{20} \text{ rad} \times 4 \text{ sec} + 0 \text{ rad}$$

$$\theta = \frac{\pi}{5} \text{ rad}$$

$$x = r \cos (\theta) = 15 \cos \left( \frac{\pi}{5} \right) \approx 12.13525492$$

$$y = r \sin (\theta) = 15 \sin \left( \frac{\pi}{5} \right) \approx 8.816778784$$

$$(x, y) \approx (12.14, 8.82)$$

(b) Give Ron’s $x$ and $y$ coordinates after 70 seconds.

$$v = 3 \text{ ft/sec} \quad r = 15 \text{ feet} \Rightarrow \omega = \frac{v}{r} = \frac{3}{15} = \frac{1}{5} \text{ rad/sec}$$

$$\theta_0 = -\frac{\pi}{2} \text{ rad}$$

$$\theta = \omega t + \theta_0 = \frac{1}{5} \text{ rad} \times 70 \text{ sec} - \frac{\pi}{2} \text{ rad}$$

$$\theta = 14 - \frac{\pi}{2} \approx 12.42920867 \text{ rad}$$

$$r = \text{lof}$$

$$x = 15 \cos \left( 14 - \frac{\pi}{2} \right) \approx 14.85911034$$

$$y = 15 \sin \left( 14 - \frac{\pi}{2} \right) \approx -2.051058273$$

$$(x, y) \approx (14.86, -2.05)$$
3. (10 points) Consider the following belt-and-wheel system with the three wheels A, B, and C. Wheel C has radius 6 inches and Wheel B has radius 4 inches. Wheel C has an angular speed of 10 revolutions/minute and Wheel A has a linear speed of 1000 inches/minute. This situation is illustrated below.

Find the radius of Wheel A.

Conversion: \( \frac{10 \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 20\pi \frac{\text{rad}}{\text{min}} \)

<table>
<thead>
<tr>
<th>( r )</th>
<th>( v )</th>
<th>( \omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1000 in/min</td>
<td>( 30\pi \frac{\text{rad}}{\text{min}} )</td>
</tr>
<tr>
<td>B</td>
<td>120\pi in/min</td>
<td>( 30\pi \frac{\text{rad}}{\text{min}} )</td>
</tr>
<tr>
<td>C</td>
<td>120\pi in/min</td>
<td>( 20\pi \frac{\text{rad}}{\text{min}} )</td>
</tr>
</tbody>
</table>

\[ \omega_B = \frac{v_B}{r_B} = \frac{120\pi}{4} = 30\pi \frac{\text{rad}}{\text{min}} \]

\[ v_c = \omega_c r_c = (20\pi) 6 = 120\pi \text{ in/min} \]

Analysis: \( v_B = v_C, \quad \omega_A = \omega_B \)

\[ r_A = \frac{v_A}{\omega_A} = \frac{1000}{30\pi} \approx 10.61032954 \text{ inches} \]

10.61 inches
4. (10 points) Grace is standing in her front yard and sees an airplane overhead. The plane is flying away from Grace at a constant height and a constant speed of 320 feet/second. When Grace sees the plane for the first time the angle measures 40 degrees. Grace sees the plane for the second time 10 seconds later and measures an angle of 30 degrees. (Hint: The plane travels 3200 feet between sightings.)

Find the height of the airplane.

\[ \tan(30^\circ) = \frac{y}{x+3200} \Rightarrow y = (x+3200) \tan(30^\circ) \]

\[ \tan(40^\circ) = \frac{y}{x} \Rightarrow y = x \tan(40^\circ) \]

Combine \[ x \tan(40^\circ) = (x + 3200) \tan(30^\circ) \]

\[ x(\tan(40^\circ) - \tan(30^\circ)) = 3200 \tan(30^\circ) \]

\[ x = \frac{3200 \tan(30^\circ)}{\tan(40^\circ) - \tan(30^\circ)} \approx 7058.356604 \text{ feet} \]

\[ y = x \tan(40^\circ) = \frac{3200 \tan(30^\circ) \tan(40^\circ)}{\tan(40^\circ) - \tan(30^\circ)} \]

\[ \approx 5922.666102 \text{ feet} \]

height \approx 5922.67 \text{ feet}
5. (10 points)

a) Let \( g(x) = \begin{cases} 
  x + 2, & \text{if } x < 7 \\
  13, & \text{if } x \geq 7 
\end{cases} \) and \( h(x) = \begin{cases} 
  -6, & \text{if } x < 3 \\
  x - 5, & \text{if } x \geq 3 
\end{cases} \).

Find the multipart rule for \( 2g(x) - h(x) \).

\[
2g(x) - h(x) = \begin{cases} 
  2(x + 2) - (-6), & \text{if } x < 3 \\
  2(x + 2) - (x - 5), & \text{if } 3 \leq x < 7 \\
  2(13) - (x - 5), & \text{if } x \geq 7 
\end{cases}
\]

b) Let \( f(x) = \frac{x}{x + 8} \).

Find \( f^{-1}(x) \) and give the domain of \( f^{-1}(x) \).

\[
y = \frac{x}{x + 8} \quad \text{and} \quad yx + 8y = x \\
yx = (1-y)x \\
x = \frac{8y}{1-y}
\]

\[
f^{-1}(x) = \frac{8x}{1-x} = \frac{-8x}{x-1} \quad \frac{\text{Domain}}{x \neq 1}
\]