1. (12 pts) Use derivatives and anti-derivatives to compute the following:

(a) Let $f(x) = \dfrac{\ln(18x+3)}{4x+2}$. Find the slope of the tangent line to $f(x)$ at $x = 0$.
(Give your answer accurate to 3 digits after the decimal).

$$f'(x) = \frac{(4x+2)\frac{18}{18x+3} - \ln(18x+3)\cdot 4}{(4x+2)^2}$$

$$f'(0) = \frac{2\cdot\frac{18}{3} - \ln(3)\cdot 4}{(2)^2} = \frac{12-4\ln(3)}{4} = 3-\ln(3) \approx 1.901$$

$\boxed{f'(0) = 3-\ln(3) \approx 1.901}$

(b) Let $TC(x) = 50 + 6x^2e^{x/2}$ dollars where x is in items. Find the marginal cost at $x = 2$ items.
(Round your answer to the nearest cent).

$$TC'(x) = 3x^2e^{x/2} + 12xe^{x/2}$$

$$TC'(2) = 12e^1 + 24e^1 = 36e \approx 97.858$$

$\boxed{MC(2) = 36e \approx 97.858}$

(c) Find the general anti-derivative: $\displaystyle\int \frac{3}{\sqrt{x^3}} + \frac{2}{5x}\,dx$. Put a box around your final answer.

$$= \int 3x^{-3/2} + \frac{2}{5}\frac{1}{x}\,dx$$

$$= 3\frac{1}{-1/2}x^{-1/2} + \frac{2}{5}\ln(x) + c = \boxed{-6x^{-1/2} + \frac{2}{5}\ln(x) + c}$$

(d) Evaluate $\displaystyle\int_0^1 x^2(8x-3) + 6e^{2x}\,dx$. Put a box around your final answer.

$$\int_0^1 8x^3 - 3x^2 + 6e^{2x}\,dx = 2x^4 - x^3 + 3e^{2x}\Big|_0^1$$

$$= (2-1+3e^2) - (0-0+3)$$

$$= 1 + 3e^2 - 3$$

$$= \boxed{3e^2 - 2 \approx 20.167}$$

2. (13 pts) You sell Things. The functions for marginal revenue and average cost (both in dollars/item) are given by

$$MR(q) = 50 - 2q \text{ and } AC(q) = \frac{20}{q} + 2 + q, \; = 20q^{-1} + 2 + q$$

where q is in **thousands** of items.
Keep enough digits to be accurate to the nearest Thing and nearest dollar.

(a) Is **Total Revenue** concave up, concave down, or neither at $q = 4$ items?
(Show some work/calculations to justify your answer)

$TR'(q) = 50 - 2q$

$TR''(q) = -2$ NEGATIVE (EVERYWHERE, INCLUDING 4)

Circle One: CONCAVE UP or $\boxed{\text{CONCAVE DOWN}}$ or NEITHER

(b) Find the one positive critical value for Average Cost and use either the 1st derivative number line or the second derivative test to determine if it gives a local maximum, local minimum, or neither (clearly show your reasoning).

$AC'(q) = -20q^{-2} + 1 \overset{?}{=} 0 \Rightarrow -\frac{20}{q^2} + 1 = 0 \Rightarrow q^2 = 20 \Rightarrow q = \sqrt{20}$
≈ 4.472

1st deriv. numberline

AC ↘ | ↗

$AC' < 0$ $\sqrt{20}$ $AC' > 0$

OR

$AC'' = 40q^{-3}$ POSITIVE (CONCAVE UP)

The critical point $q =$ 4.472 thousand Things gives a

(CIRLCE ONE): $\boxed{\text{LOCAL MIN}}$ or LOCAL MAX or NEITHER

(c) Find the maximum profit.

$TR(q) = 50q - q^2$, $TC(q) = 20 + 2q + q^2 \Rightarrow MC(q) = 2 + 2q$

$MR = MC \Rightarrow 50 - 2q \overset{?}{=} 2 + 2q \Rightarrow 48 = 4q \Rightarrow q = 12$

$P(12) = TR(12) - TC(12)$
$= (50(12) - (12)^2) - (20 + 2(12) + (12)^2)$
$= 456 - 188 = 268$

$\boxed{268}$ thousand dollars

3. (12 pts) The amount of water in two vats is changing. The amount of water (in gallons) in Vat A and in Vat B are given by $A(t)$ and $B(t)$ respectively, where t is in hours. You are told that the vats start with the same amount of water and that

Vat A RATE of change:	$A'(t) = -3t^2 + 18t - 15$	gallons/hour
Vat B AMOUNT:	$B(t) = -t^2 + 14t + 50$	gallons

(a) Find the formula for $A(t)$ without any undetermined constants.
(Hint: the problem told you $A(0) = B(0)$).

$A(t) = -t^3 + 9t^2 - 15t + C$

$A(0) = B(0) = 50$

$$\boxed{A(t) = -t^3 + 9t^2 - 15t + 50}$$

(b) Find all times at which $A(t)$ has a point of inflection. (Justify your answer by drawing the 2nd deriv. number line, indicating concavity, as we have done in class).

$A'(t) = -3t^2 + 18t - 15$

$A''(t) = -6t + 18 \stackrel{?}{=} 0 \Rightarrow \boxed{t = 3}$

$\cup$ | $\cap$

$A'' > 0$ 3 $A'' < 0$

$$\boxed{t = 3 \text{ hours}}$$

(c) What is the highest *amount* in Vat A during the interval from $t = 0$ to $t = 7$ hours?

$A'(t) = -3t^2 + 18t - 15 \stackrel{?}{=} 0 \Rightarrow -3(t^2 - 6t + 5) \stackrel{?}{=} 0$

$-3(t-1)(t-5) = 0$

$t = 1, t = 5$

$A(0) = 50$

$A(1) = 43$

$A(5) = 75$

$A(7) = 43$

$$\boxed{75 \text{ gallons}}$$

(d) What is the highest *rate* of change in Vat B on the interval $t = 0$ to $t = 7$? (*i.e.* level is rising most rapidly)

$B'(t) = -2t + 14$ ← WANT MAX OF THIS!

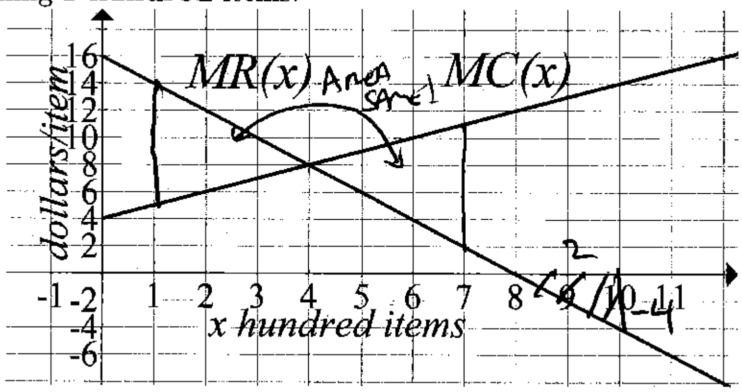
$-2 \stackrel{?}{=} 0$ ← NEVER (NO CRITICAL PTS)

$B'(0) = 14$

$B'(7) = 0$

$$\boxed{14 \text{ gallons/hour}}$$

4. (13 pts) The graph below shows marginal revenue and marginal cost (in dollars per item) for producing and selling x **hundred** items.



You are also told that **Fixed Costs** are $FC = \$1050$ (10.5 hundred dollars). Use the picture to estimate the answers to the questions below *as accurately as possible.*

(a) For the 3 quick questions below, fill in the blanks:

i. Total Revenue is maximized at $x =$ 8 hundred items

ii. Marginal Revenue is maximized at $x =$ 0 hundred items

iii. Profit is maximized at $x =$ 4 hundred items

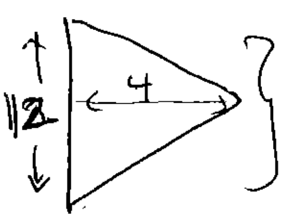
(b) Estimate the following from the graph:

i. $\int_8^{10} MR(x)\,dx = \frac{1}{2}(2)(-4) = \boxed{-4}$ hundred dollars

ii. $TC''(3) = MC'(3) =$ "SLOPE OF MC AT 3" $= \frac{8-4}{4-0} = \frac{4}{4} = \boxed{1}$

A LINE! (0,4) (4,8)
USE 2 PTS & GET SLOPE!

(c) Estimate the maximum profit.



Area $= \frac{1}{2}(12)(4) = 24$

$P(4) = \underbrace{P(0)}_{-10.5} + 24 = 13.5$

Max Profit = 13.5 hundred dollars

(d) There are two quantities when profit is zero. Find them both. (Hint: Think very carefully, take your time, and remember that profit starts at -10.5 hundred dollars)

AREA FROM 0 TO 1 = "5 + 1/4 BOXES" ≈ 10.5 HUNDRED DOLLARS

So $P(1) = \underbrace{P(0)}_{-10.5} + 10.5 = 0$

THEN AGAIN AT 7 (MATCH AREAS)

$x \approx$ 1 and $x \approx$ 7 hundred items