1. (12 points)

(a) For the two derivative questions below, you do NOT have to simplify your final answer. *Put a box around your final answers.*

i. Find $f'(x)$, if $f(x) = 3\left(\frac{4}{x^2} + \frac{x^2}{7}\right)^5$. $= 3\left(4x^{-2} + \frac{1}{7}x^2\right)^5$

$$\boxed{f'(x) = 15\left(4x^{-2} + \frac{1}{7}x^2\right)^4 \cdot \left(-8x^{-3} + \frac{2}{7}x\right)}$$

ii. Find $\frac{dy}{dx}$, if $y = \frac{2x}{5} + \sqrt[3]{x}\sqrt{x^4 - 6x^3}$. $= \frac{2}{5}x + x^{1/3}(x^4 - 6x^3)^{1/2}$

$$\boxed{\frac{dy}{dx} = \frac{2}{5} + \frac{1}{2}x^{1/3}(x^4 - 6x^3)^{-1/2}(4x^3 - 18x^2) + \frac{1}{3}x^{-2/3}(x^4 - 6x^3)^{1/2}}$$

(b) Write the *equation of the tangent line* to the graph of $y = \frac{6x^5 - 6x - 18}{4 - x^2}$ at $x = 1$. Simplify your final answer into the form $y = mx + b$.

$$y(1) = \frac{6 - 6 - 18}{4 - 1} = \frac{-18}{3} = -6$$ ← HEIGHT

$$y' = \frac{(4 - x^2)(30x^4 - 6) - (6x^5 - 6x - 18)(-2x)}{(4 - x^2)^2}$$

SLOPE

$$y'(1) = \frac{3 \cdot 24 - (-18)(-2)}{(3)^2} = \frac{72 - 36}{9} = \frac{36}{9} = 4$$

$$\boxed{y = 4(x - 1) - 6}$$

ANSWER: $y = 4x - 10$

2. (11 pts) **Parts (a) and (b) below are NOT related.**

(a) Let $f(x) = 5x - 2x^2$.

Write out, expand and *completely simplify* the following: $\dfrac{f(x+h) - f(x)}{h}$.

Then **also** give $f'(x)$. (Feel free to check your work!)

$$\frac{[5(x+h) - 2(x+h)^2] - [5x - 2x^2]}{h}$$

$$= \frac{5x + 5h - 2(x^2 + 2xh + h^2) - \cancel{5x} + 2x^2}{h}$$

$$= \frac{5h - 2x^2 - 4xh - 2h^2 + \cancel{2x^2}}{h}$$

$$= 5 - 4x - 2h$$

ANSWERS: $\frac{f(x+h)-f(x)}{h}$ = $5 - 4x - 2h$ $\quad h \to 0$

$f'(x)$ = $5 - 4x$

(b) For a different function, $g(x)$, you are told that $g(x+h) - g(x) = h^3 + 3h^2x + 3hx^2 - h$ for all values of x and h (you are NOT given $g(x)$). Answer the following questions:

i. Give the value of $g(3) - g(2)$.

$x = 2,\ h = 1$

$$\Rightarrow g(2+1) - g(2) = (1)^3 + 3(1)^2(2) + 3(1)(2)^2 - (1)$$
$$= 1 + 6 + 12 - 1 = 18$$

ANSWER: $g(3) - g(2)$ = 18

ii. Give the value of $g'(5)$.

$$\frac{g(x+h) - g(x)}{h} = h^2 + 3hx + 3x^2 - 1$$

$$h \to 0 \Rightarrow g'(x) = 3x^2 - 1 \Rightarrow g'(5) = 3(5)^2 - 1 = 75 - 1$$

ANSWER: $g'(5)$ = 74

3. (12 pts) You sell Items. If you sell q hundred Items, you are given:

demand curve (*i.e.* price):	$p = 85 - 2q$	dollars/Item
total cost:	$TC(q) = q^3 - 20q^2 + 145q + 25$	hundred dollars

Note: **Pay attention to units.**

(a) Find the quantity and price that correspond to maximum **total revenue** (round to the nearest Item and dollar/Item)

$TR(q) = 85q - 2q^2$

$MR(q) = 85 - 4q \stackrel{?}{=} 0$

$85 = 4q$

$q = \frac{85}{4} = 21.25$ hundred Items

$p = 85 - 2(21.25) = 42.5$

ANSWER: Quantity: 2,125 Items

Price: 42.50 dollars/Item

(b) Find the longest interval on which marginal revenue exceeds marginal cost.

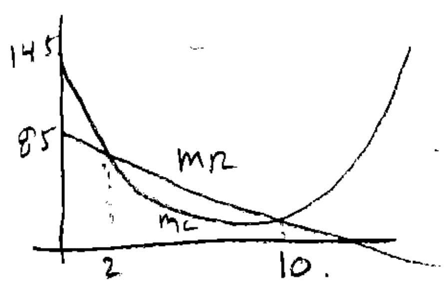
$MR(q) \stackrel{?}{=} MC(q)$.

$85 - 4q \stackrel{?}{=} 3q^2 - 40q + 145$

$0 = 3q^2 - 36q + 60$

$0 = q^2 - 12q + 20$

$0 = (q-2)(q-10)$



ANSWER: From q = 2 to q = 10 hundred Items

(c) What is the maximum value of **profit** to the nearest dollar?

OCCURS AT QUANTITY WHERE MR > MC SWITCHES TO MR < MC WHICH IS $q = 10$ FROM PART (b)

$P(10) = TR(10) - TC(10)$

$= (85(10) - 2(10^2)) - ((10)^3 - 20(10)^2 + 145(10) + 25)$

$= (650) - (475)$

$= 175$ hundred dollars

ANSWER: max profit = 17,500 dollars

4. (15 pts) **Parts (a) and (b) below are NOT related.**

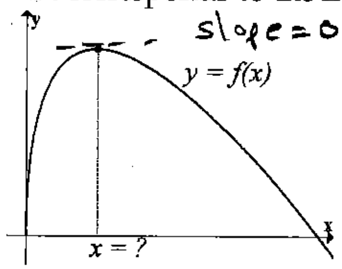
(a) Let $f(x) = 5\sqrt{x} - 4x$.

i. Find the second derivative of $f(x)$.

$f'(x) = \frac{5}{2}x^{-1/2} - 4$

ANSWER: $f''(x) = \boxed{-\frac{5}{4}x^{-3/2}}$

ii. The graph of $f(x) = 5\sqrt{x} - 4x$ is below. Use a derivative to find the x-coordinate that corresponds to the maximum point shown on the graph (shown below).



slope = 0 $\Rightarrow \frac{5}{2}x^{-1/2} - 4 \overset{?}{=} 0$

$\frac{5}{2\sqrt{x}} - 4 \overset{?}{=} 0$

$\frac{5}{2\sqrt{x}} = 4$

$5 = 8\sqrt{x}$

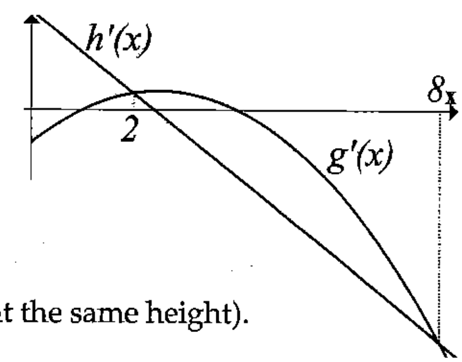
$\frac{5}{8} = \sqrt{x}$

ANSWER: $x = \boxed{\frac{25}{64}}$ $= 0.390625$

(b) Two functions $g(x)$ and $h(x)$ have derivatives

$$g'(x) = -x^2 + 5x - 4 \text{ and } h'(x) = -5x + 12.$$

The *derivative* graphs are shown, including the locations where they intersect each other (2 and 8). Note that the formulas for $g(x)$ and $h(x)$ are not given.



i. Assume $g(0) = h(0)$ (*i.e.* original functions start at the same height). For each part, circle the true statement:

A. Circle one: $h(1) > h(0)$ or $h(1) < h(0)$ or $h(1) = h(0)$.

B. Circle one: $g(2) > h(2)$ or $g(2) < h(2)$ or $g(2) = h(2)$.

ii. Name the longest interval over which $g(x)$ is increasing and $h(x)$ is increasing.

$g'(x) \overset{?}{=} 0 = -x^2 + 5x - 4$

$x^2 - 5x + 4 \overset{?}{=} 0$

$(x-1)(x-4) = 0$

g' IS POSITIVE FROM x = 1 TO x = 4

h' IS POSITIVE BEFORE x = 2.4

BOTH?

$h'(x) \overset{?}{=} 0 = -5x + 12$

$x = \frac{12}{5} = 2.4$

ANSWER: from $x =$ 1 to $x =$ 2.4