1. (12 points)

(a) For the two derivative questions below, you do NOT have to simplify your final answer. *Put a box around your final answers.*

i. Find $f'(x)$, if $f(x) = 5\left(\frac{4}{x^2} + \frac{x^2}{3}\right)^4$ $= 5\left(4x^{-2} + \frac{1}{3}x^2\right)^4$

3

$$\boxed{f'(x) = \underbrace{20\left(4x^{-2}+\tfrac{1}{3}x^2\right)^3}_{+1}\underbrace{\left(-8x^{-3}+\tfrac{2}{3}x\right)}_{+2}}$$

ii. Find $\frac{dy}{dx}$, if $y = \frac{3x}{8} + \sqrt[3]{x}\sqrt{x^7 - 4x^3}$. $= \frac{3}{8}x + x^{1/3}\left(x^7-4x^3\right)^{1/2}$

4

$$\boxed{\frac{dy}{dx} = \underbrace{\frac{3}{8}}_{+1} + \underbrace{\frac{1}{2}x^{1/3}\left(x^7-4x^3\right)^{-\frac{1}{2}}\left(7x^6-12x^2\right)}_{+2} + \underbrace{\frac{1}{3}x^{-2/3}\left(x^7-4x^3\right)^{1/2}}_{+1}}$$

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(b) Write the *equation of the tangent line* to the graph of $y = \frac{3x^5 - 3x - 9}{4 - x^2}$ at $x = 1$. Simplify your final answer into the form $y = mx + b$.

$$y(1) = \frac{3-3-9}{4-1} = \frac{-9}{3} = -3 \leftarrow \text{HEIGHT}$$

$$+2 \left\{ y' = \frac{(4-x^2)(15x^4-3) - (3x^5-3x-9)(-2x)}{(4-x^2)^2} \right.$$

$$+1 \quad y'(1) = \frac{3\cdot 12 - (-9)(-2)}{(3)^2} = \frac{36-18}{9} = \frac{18}{9} = 2 \leftarrow \text{SLOPE}$$

$+2$ $\left\{\right.$

$$\boxed{y = 2(x-1) - 3}$$

ANSWER: $y =$ $\boxed{2x - 5}$

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2. (11 pts) **Parts (a) and (b) below are NOT related.**

   (a) Let $f(x) = 3x - 2x^2$.

   Write out, expand and *completely simplify* the following: $\dfrac{f(x+h) - f(x)}{h}$.

   Then **also** give $f'(x)$. (Feel free to check your work!)

$$\frac{[3(x+h) - 2(x+h)^2] - [3x - 2x^2]}{h}$$ +2

$$= \frac{\cancel{3x} + 3h - 2(x^2 + 2xh + h^2) - \cancel{3x} + 2x^2}{h}$$

+2

$$= \frac{3h - \cancel{2x^2} - 4xh - 2h^2 + \cancel{2x^2}}{h}$$

$$= 3 - 4x - 2h$$

ANSWERS: $\frac{f(x+h)-f(x)}{h}$ = $3-4x-2h$ $h \to 0$

+1 $f'(x)$ = $3-4x$

   (b) For a different function, $g(x)$, you are told that $g(x+h) - g(x) = 2h^3 + 6h^2x + 6hx^2 - h$ for all values of x and h (you are NOT given $g(x)$). Answer the following questions:

   i. Give the value of $g(3) - g(2)$.

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$x = 2, h = 1$

$$\Rightarrow g(2+1) - g(2) = 2(1)^3 + 6(1)^2(2) + 6(1)(2)^2 - (1)$$

$$= 2 + 12 + 24 - 1 = 37$$

ANSWER: $g(3) - g(2)$ = $\boxed{37}$

   ii. Give the value of $g'(5)$.

3

$$\frac{g(x+h) - g(x)}{h} = 2h^2 + 6hx + 6x^2 - 1$$

$$h \to 0 \Rightarrow g'(x) = 6x^2 - 1 \Rightarrow g'(5) = 6(5)^2 - 1 = 150 - 1$$

+2

ANSWER: $g'(5)$ = $\boxed{149}$

3. (12 pts) You sell Items. If you sell q hundred Items, you are given:

demand curve (*i.e.* price):	$p = 81 - 2q$	dollars/Item
total cost:	$TC(q) = q^3 - 20q^2 + 141q + 2$	hundred dollars

Note: Pay attention to units. +1 UNITS

4

(a) Find the quantity and price that correspond to maximum **total revenue** (round to the nearest Item and dollar/Item)

$$TR(q) = 81q - 2q^2$$
$$MR(q) = 81 - 4q \stackrel{?}{=} 0$$
$$81 = 4q$$
$$q = \frac{81}{4} = 20.25 \text{ hundred Items}$$
$$p = 81 - 2(20.25) = 40.5$$

+3

+1

ANSWER: Quantity: 2,025 Items

Price: 40.50 dollars/Item

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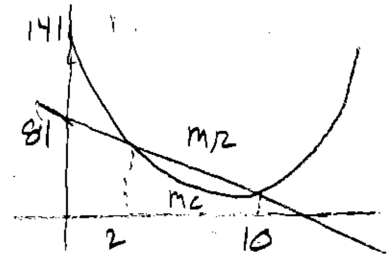
(b) Find the longest interval on which marginal revenue exceeds marginal cost.

$$MR(q) \stackrel{?}{=} MC(q)$$
$$81 - 4q \stackrel{?}{=} 3q^2 - 40q + 141$$
$$0 = 3q^2 - 36q + 60$$
$$0 = q^2 - 12q + 20$$
$$0 = (q-2)(q-10)$$

+3



ANSWER: From q = 2 to q = 10 hundred Items +1

3

(c) What is the maximum value of **profit** to the nearest dollar?

OCCURS AT QUANTITY WHERE MR > MC SWITCHES TO MR < MC WHICH IS $q = 10$ FROM PART (b).

$$P(10) = TR(10) - TC(10)$$
$$= \left(81(10) - 2(10)^2\right) - \left((10)^3 - 20(10)^2 + 141(10) + 2\right)$$
$$= (610) - (412)$$
$$= 198 \text{ hundred dollars}$$

ANSWER: max profit = 19,800 dollars

4. (15 pts) **Parts (a) and (b) below are NOT related.**

(a) Let $f(x) = 3\sqrt{x} - 4x$.

i. Find the second derivative of $f(x)$.

$f'(x) = \frac{3}{2}x^{-1/2} - 4$

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ANSWER: $\boxed{f''(x) = -\frac{3}{4}x^{-3/2}}$

ii. The graph of $f(x) = 3\sqrt{x} - 4x$ is below. Use a derivative to find the x-coordinate that corresponds to the maximum point shown on the graph (shown below).

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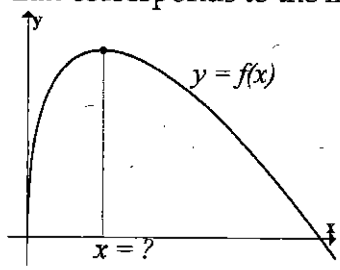


$\frac{3}{2}x^{-1/2} - 4 \stackrel{?}{=} 0$

$\frac{3}{2\sqrt{x}} - 4 \stackrel{?}{=} 0$

$\frac{3}{2\sqrt{x}} = 4$

$3 = 8\sqrt{x}$

$\rightarrow \sqrt{x} = \frac{3}{8}$

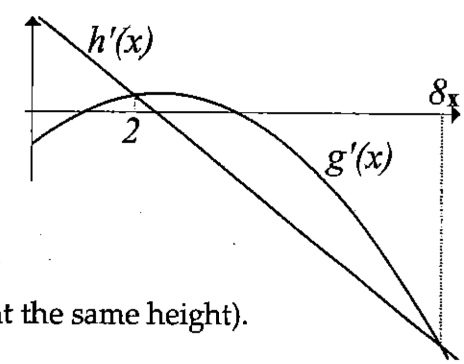
$x = \frac{9}{64}$

ANSWER: $\boxed{x = \frac{9}{64}}$ ≈ 0.140625

(b) . Two functions $g(x)$ and $h(x)$ have derivatives

$$g'(x) = -x^2 + 5x - 4 \text{ and } h'(x) = -5x + 12.$$

The *derivative* graphs are shown, including the locations where they intersect each other (2 and 8). Note that the formulas for $g(x)$ and $h(x)$ are not given.



i. Assume $g(0) = h(0)$ (*i.e.* original functions start at the same height). For each part, circle the true statement:

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A. Circle one: $g(2) > h(2)$ or $g(2) = h(2)$ or $g(2) < h(2)$.

B. Circle one: $h(1) > h(0)$ or $h(1) = h(0)$ or $h(1) < h(0)$.

ii. Name the longest interval over which $g(x)$ is increasing and $h(x)$ is decreasing.

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$g'(x) \stackrel{?}{=} 0 = -x^2 + 5x - 4$

$x^2 - 5x + 4 = 0$

$(x-1)(x-4) = 0$

$\Rightarrow$ $g'(x)$ positive from $x = 1$ to $x = 4$

$h'(x)$ negative from $\underbrace{x = 2.4 \text{ ONWARD}}_{\text{BOTH ?}}$

$h'(x) = 0 = -5x + 12$

$x = \frac{12}{5} = 2.4$

ANSWER: $\boxed{\text{from } x = 2.4 \text{ to } x = 4}$