MR/MC Overview and Redefinition

Summary of Math 111 Definitions: In Math 111, you worked extensively with the terms of business. You should be very comfortable with all of the following:

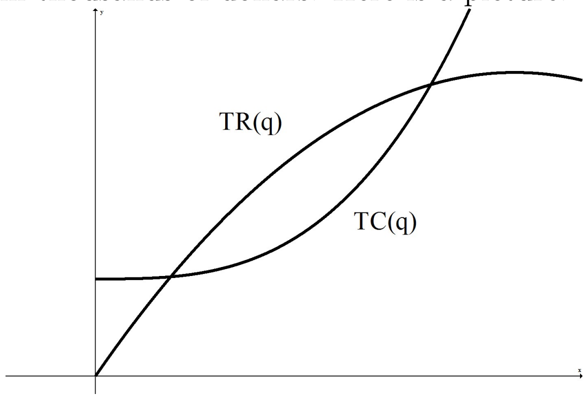
1. p = price per unit
2. q = quantity produced and sold (watch your units this is ofen in hundreds or thousands).
3. $TR(q)$ = Total Revenue = 'the total amount of money that comes in from selling q units.'
   Note that typically $TR(q) = pq$ = (price)(quantity), and often we know a demand function that gives q in terms of p (which we can solve for p in terms of q).
4. $TC(q)$ = Total Cost = 'the total amount of cost to produce q units.'
5. $FC = TC(0)$ = Fixed Cost = 'the costs that don't depend on production level'
   = 'the cost you still must pay even if you produce zero units'
6. $VC(q) = TC(q) - FC$ = Variable Cost = 'the costs that depend only on the number of units produced'
   We often write $TC(q) = VC(q) + FC$.
7. $P(q) = TR(q) - TC(q)$ = Profit = 'money remaining when cost is subtracted from revenue'
8. $AC(q) = \frac{TC(q)}{q}$ = Average Cost = 'the overall average cost per unit'
9. $AVC(q) = \frac{VC(q)}{q}$ = Average Variable Cost = 'the overall average of variable cost per unit'
10. $AR(q) = \frac{TR(q)}{q}$ = Average Revenue = 'the overall revenue per unit'
    (Note: since $TR(q) = pq$ and $AR(q) = \frac{TR(q)}{q}$, $AR(q)$ is the same as price!).
11. $MR(q)$ = Marginal Revenue = 'the revenue that the next item will bring in'
12. $MC(q)$ = Marginal Revenue = 'the cost required to produce the next item'

Redefinition of MR/MC: We have started to see that we can quickly find the formula for the slope of the tangent (*e.g.* the derivative). Also there are many nice properties of the derivative that we can use in general to analyze and optimize functions. For these reasons, in Math 112, we redefine MR and MC as follows:

1. $MR(q) = TR'(q)$ = 'the slope of the tangent line to $TR(q)$'
2. $MC(q) = TC'(q)$ = 'the slope of the tangent line to $TC(q)$'

This is NOT exactly the same as the Math 111 definition, but it is very, very close to the same and it is standard to interpret it in the same way as before. The previous definition was talking about a change in height on the graph and since that change was only one unit, you found in Math 111 that this was the same as the slope of the secant line over a one unit interval. The derivative is the slope of tangent line. When you look at a graph typically the slope of the tangent at q and the slope of the secant from q to q+'one unit' are very close to the same thing (especially if the units are in hundreds or thousands). It is good to understand that there is a slight difference in definitions, but, for this class, we will use the derivative as the definition and we will still interpret the results the way we did in Math 111.

Example: Let's say that $TR(q) = 10q - 2q^2$ and $TC(q) = q^3 + 4$. Also, assume that q is in thousands of items and TC and TR are in thousands of dollars. Here is a picture:



Just for quick review here are the business functions reviewed on the previous page (these haven't changed from Math 111):

- $P(q) = TR(q) - TC(q) = 10q - 2q^2 - q^3 - 4 = -q^3 - 2q^2 + 10q - 4$ thousand dollars
- $FC = TC(0) = 4$ thousand dollars
- $VC(q) = TC(q) - FC = q^3$ thousand dollars

  Note about the units for all slopes (averages and derivatives): When you divide thousands of dollars by thousands of units, it is the same as dollars per unit).

- $AC(q) = \frac{TC(q)}{q} = \frac{q^3+4}{q} = q^2 + \frac{4}{q}$ dollars per unit
- $AVC(q) = \frac{VC(q)}{q} = q^2$ dollars per unit
- $AR(q) = \frac{TR(q)}{q} = \frac{10q-2q^2}{q} = 10 - 2q$ dollars per unit
  Note: From this last one, we see that $p = 10 - 2q$ is the price formula (or equivalently $q = \frac{10-p}{2}$ is the demand function).

Here are MR and MC (using the new Math 112 definition):

- $MR(q) = TR'(q) = 10 - 4q$ dollars per item
- $MC(q) = TC'(q) = 3q^2$ dollars per item

Let's just analyze what is happening at a particular quantity. How about $q = 1$ thousand items. If you sell and produce $q = 1$ thousand items, then

1. The price per item is $p(1) = 10 - 2(1) = 8$ dollars per item.
   The money brought in is $TR(1) = 10(1) - 2(1)^2 = 8$ thousand dollars.
   The cost to produce is $TC(1) = (1)^3 + 4 = 5$ thousand dollars (4 thousand of which is fixed costs and the rest is variable cost).
   Thus, at $q = 1$ thousand items, then profit is $P(1) = TR(1) - TC(1) = 3$ thousand dollars.

2. $MR(1) = 10 - 4(1) = 6$ dollars per item (so if you sell one more item (from $q = 1$ to $q = 1.001$), then your revenue will go up about 6 dollars).
   $MC(1) = 3(1)^2 = 3$ dollars per item (so if you sell one more item, then your cost will go up 3 dollars).
   The conclusion from Math 111, which still holds, is

   - Whenever $MR > MC$, then profit is going up (you want to sell another item because the revenue will exceed cost).
   - Whenever $MR < MC$, then profit is going down (you don't want to sell another item because the cost will exceed revenue).
   - When you switch from $MR > MC$ to $MR < MC$, you have maximum profit! So you need to find when $MR = MC$ to find max profit.

   In this example, you would be solving $MR(q) = MC(q)$, which gives the equation

   $$10 - 4q = 3q^2$$

   Getting everything to one side and using the quadratice formula gives: $q = -2.6103$ and $q = 1.2770$. So the maximum profit occurs when you produce and sell $q = 1.277$ thousand items.