9.7 and 9.8 Overview

My reviews and review sheets are not meant to be your only form of studying. It is vital to your success on the exams that you carefully go through and understand ALL the homework problems, worksheets and lecture material. Hopefully this review sheet will remind you of some of the key ideas of these sections.

9.7: Summary/Combining of Derivative Rules

- Here is a shorten version of the steps to differentiate:
  1. Simplify and Rewrite powers (write roots are fractional exponents and use negative exponents). The goal here is make your starting expression easier to work with.
  2. Sums and Coefficients come along for the ride. Included in this steps you should identity what is a coefficent (For example, the expression $\frac{4(2x+1)^{10}}{7}$ should be rewritten as $\frac{4}{7}(2x+1)^{10}$, so that you can see that $\frac{4}{7}$ is just a coefficient).
  3. Identify the form:
     (a) Product Rule? If in the form $First \cdot Second = F \cdot S$, then use $(F \cdot S)' = F \cdot S' + S \cdot F'$.
     (b) Quotient Rule? If in the form $\frac{Numerator}{Denominator} = \frac{N}{D}$, then use $\left(\frac{N}{D}\right)' = \frac{D \cdot N' - N \cdot D'}{D^2}$.
     (c) Chain Rule? If in the form $(f(x))^n$, then use $((f(x))^n)' = n(f(x))^{n-1} \cdot f'(x)$.
  4. In the process of using these rules, you may have to do other derivatives. In which case start this analysis over on those sub derivatives and put the results back in the correct location in the previous rule. Be organized!

- You have already seen dozens and dozens of examples in lecture, in the homework, and posted online, but here are yet a couple more to illustrate the idea:
  1. $f(x) = \frac{3}{2(4x-1)^5} - 13\sqrt{x}$. Find $f'(x)$.
     (a) Simplify and Rewrite Powers to get: $f(x) = \frac{3}{2}(4x-1)^{-5} - 13x^{1/2}$.
     (b) Sums and Coefficients come along for the ride so the final answer will look like:
     $f'(x) = \frac{3}{2}??? - 13???$.
     We can already do the second term quickly so we know
     $f'(x) = \frac{3}{2}??? - 13 \cdot \frac{1}{2}x^{-1/2}$
     (c) Now we focus on how to differentiate $(4x-1)^{-5}$. This is a Chain Rule form, so the deriative is $-5(4x-1)^{-6} \cdot 4$.
     (d) And putting this all together gives

     $$f'(x) = \frac{3}{2} \cdot -5(4x-1)^{-6} \cdot 4 - 13 \cdot \frac{1}{2}x^{-1/2} = -30(4x-1)^{-6} - \frac{13}{2}x^{-1/2}.$$

     You might notice that at the very end I simplified by multiplying out all the constants in front of each term ($-30 = \frac{3}{2} \cdot (-5) \cdot 4$ and $\frac{13}{2} = 13 \cdot \frac{1}{2}$). This sure makes the expressions easier to work with and look at. You should get in the practice of doing this easy simplification at the end of your problems.

2. $g(x) = 3(x^2+4x)^2 \cdot (\frac{4}{x^3}+1)^8$. Find $g'(x)$.
   (a) Simplify and Rewrite Powers to get: $g(x) = 3(x^2+4x)^2 \cdot (4x^{-3}+1)^8$.
   (b) Sums and Coefficients come along for the ride so the final answer will look like: $f'(x) = 3?????????$.
   (c) Now we focus on how to differentiate $(x^2+4x)^2 \cdot (4x^{-3}+1)^8$. This is a Product Rule form, so the derivative will look like:
   $g'(x) = 3[(x^2+4x)^2 \cdot ??? + (4x^{-3}+1)^8 \cdot ???]$
   (d) To fill in the last spots, we need to know the derivatives of $S = (4x^{-3}+1)^8$ and $F = (x^2+4x)^2$. Those are chain rule forms. So we get $S' = 8(4x^{-3}+1)^7 \cdot -12x^{-4}$ and $F' = 2(x^2+4x)\cdot(2x+4)$. Putting it all together gives:

$$g'(x) = 3[(x^2+4x)^2 \cdot 8(4x^{-3}+1)^7 \cdot -12x^{-4} + (4x^{-3}+1)^8 \cdot 2(x^2+4x)\cdot(2x+4)],$$

   multiplying out the coefficients in each term and distributing the 3 gives $3 \cdot 8 \cdot -12 = -288$ and $3 \cdot 2 = 6$, so we can quickly simplify to

$$g'(x) = -288(x^2+4x)^2 \cdot (4x^{-3}+1)^7 \cdot x^{-4} + 6(4x^{-3}+1)^8 \cdot (x^2+4x)\cdot(2x+4).$$

   Aside: If you want to practice making it look even nicer (not required for the midterm), the next thing you can do is look for things in common and factor them out. The idea of factoring is simple. For example, since $5x^2(x^3+1) = 5x^5+5x^2$, if you ever see $5x^5+5x^2$, then you can notice that both terms have $5x^2$ in common so you can divide that out of each term and put it in front to get $5x^2(x^3+1)$. So the idea is you look for everything that is in common in each term, you divide that commonality out of each term, and you write it as a factor out in front.
   In this case, we see that both terms of $g'(x)$ have $(x^2+4x)^1$ and $(4x^{-3}+1)^7$. If we divide (cancel) these out of both terms and write them out in front we get

$$g'(x) = (x^2+4x)(4x^{-3}+1)^7(-288(x^2+4x)\cdot x^{-4} + 6(4x^{-3}+1)\cdot(2x+4)).$$

   Again, this sort of simplifying is NOT required on the tests, but a lot of students have been asking about factoring, so I wanted to give one example.

9.8: Second Derivatives

- The second derivative is the derivative of the derivative. It is the rate of change of the rate of change (it tells us how quickly the slopes of the tangent line to the original graph are changing). The second derivative of a distance formula is the rate of change of speed, in other words, the acceleration. The second derivative of Total Revenue is the rate of change of Marginal Revenue (it tells use how quickly Marginal Revenue is going up or down). We will discuss how to interpret the second derivative more in chapter 10 and we will use it a lot in applications. We did discuss some applications in lecture, but for now you just need to be able to compute it.

- Here is one basic example: $f(x) = x^3 + \frac{5}{3x^2} - 9\sqrt[3]{x^5}$. Find $f''(x)$.

  1. First, we rewrite to get $f(x) = x^3 + \frac{5}{3}x^{-2} - 9x^{5/3}$, then we differentiate to get $f'(x) = 3x^2 - \frac{10}{3}x^{-3} - 15x^{2/3}$. (I simplified the numbers out in front as I went, you should do the work and verify you get the same derivatives).
  2. Thus, $f''(x) = 6x + 10x^{-4} - 10x^{-1/3}$.