9.5 and 9.6 Overview

My reviews and review sheets are not meant to be your only form of studying. It is vital to your success on the exams that you carefully go through and understand ALL the homework problems, worksheets and lecture material. Hopefully this review sheet will remind you of some of the key ideas of these sections.

We now have the basic derivative rules:

NAME	RULE	EXAMPLE
Power Rule	$(x^n)' = nx^{n-1}$	$(x^{3/4})' = \frac{3}{4}x^{-1/4}$
Sum Rule	$(f+g)' = f' + g'$	$(x^5 + x^{-2})' = 5x^4 - 2x^{-3}$
Coefficient Rule	$(cf)' = cf'$	$(7x^{11})' = 7 \cdot 11x^{10} = 77x^{10}$
Product Rule	$(f \cdot g)' = f \cdot g' + g \cdot f'$	$((x^2 - x)(x^8 + x^{-1} + 4))'$ $= (x^2 - x)(8x^7 - x^{-2}) + (x^8 + x^{-1} + 4)(2x - 1)$
Quotient Rule	$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{(g)^2}$	$\left(\frac{x^2}{3x+1}\right)' = \frac{(3x+1)(2x)-(x^2)(3)}{(3x+1)^2}$
General Power Rule	$([f(x)]^n)' = n[f(x)]^{n-1} \cdot f'(x)$	$y = (x^4 - x^2)^{30}$ $\Rightarrow y' = 30(x^4 - x^2)^{29} \cdot (4x^3 - 2x)$

Here are some guiding steps to help you differentiate:

1. Clean up first:
   - Rewrite the powers: At the beginning, write roots as fractional powers and write $\frac{1}{x^a} = x^{-a}$.
   - Expand if it makes things simpler.
2. Identify the outermost operation:
   - If parts of the expressions are completely separated by sums or differences, then do those parts separately (sum/difference rule).
   - If you see two functions multiplied together, then start by using the product rule.
   - If you see two functions forming a fraction, then start by using the quotient rule.
   - If you see a function inside a power, then start by using the general power rule.
3. Set up the appropriate rule. Then compute the necessary derivatives. Be organized and don't skip any steps!

Find tangent line equations

Using the rules from the previous page, we can now quickly find the slope of the tangent line at a point for even very complicated functions. For some applications, we also want the equation for the tangent line as well. Let recall a few basic facts about lines.

1. *Slope-Intercept Form*: Many students enter calculus with a preference for the following form for a line. Suppose you know: the slope $= m$, and the y-intercept $= b$ (meaning the line goes through the point $(0, b)$). Then every point (x, y) on the line satisfies the relationship

$$y = mx + b.$$

2. *Point-Slope Form*: For calculus, it saves you some work if you instead use this form of the line. Suppose you know: the slope $= m$, and some point on the line $= (x_0, y_0)$ (any point, it doesn't have to be the y-intercept point). Then every point (x, y) on the line satisfies the relationship

$$y = m(x - x_0) + y_0.$$

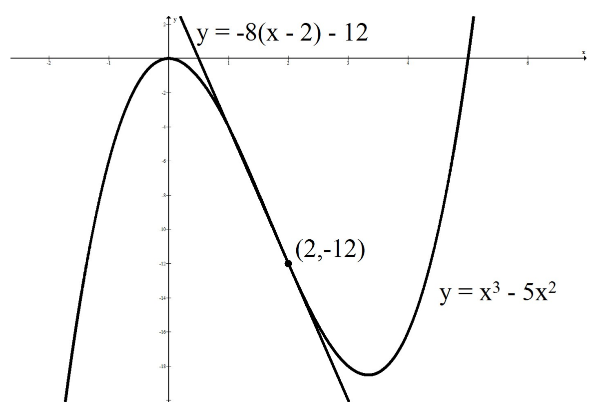
Here are some examples:

1. Find the equation for the tangent line to $f(x) = x^3 - 5x^2$ at $x = 2$.

   - The points on the graph has $x = 2$ and $y = f(2) = (2)^3 - 5(2)^2 = 8 - 20 = -12$. Thus $(x_0, y_0) = (2, -12)$.
   - The slope of the tangent is given by $f'(x) = 3x^2 - 10x$. So at $x = 2$, we get slope $= f'(2) = 3(2)^2 - 10(2) = 12 - 20 = -8$.
   - The equation for the tangent line is

$$y = -8(x - 2) - 12.$$

Here is a visual of what we just found:

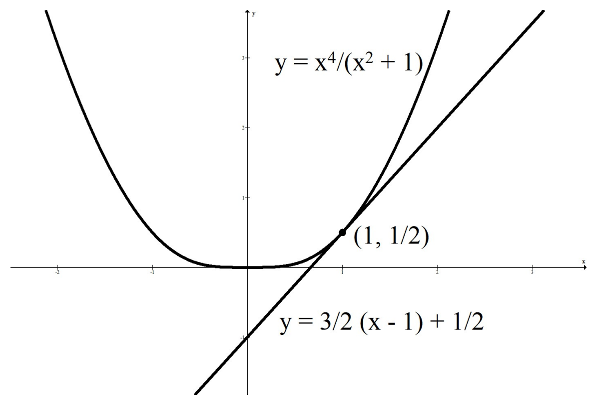


2. Find the equation for the tangent line to $f(x) = \frac{x^4}{x^2+1}$ at $x = 1$.

- The points on the graph has $x = 1$ and $y = f(1) = \frac{(1)^4}{(1)^2+1} = \frac{1}{2}$. Thus, $(x_0, y_0) = \left(1, \frac{1}{2}\right)$.
- The slope of the tangent is given by $f'(x) = \frac{(x^2+1)(4x^3)-x^4(2x)}{(x^2+1)^2}$. So at $x = 1$, we get slope $= f'(1) = \frac{((1)^2+1)(4(1)^3)-(1)^4(2(1))}{((1)^2+1)^2} = \frac{8-2}{4} = \frac{6}{4} = \frac{3}{2}$.
- The equation for the tangent line is

$$y = \frac{3}{2}(x-1) + \frac{1}{2}.$$

Again, here is a picture of what we just found:



3. Find the equation for the tangent line to $f(x) = (x^2+3)\sqrt{5x+1}$ at $x = 0$.

- The points on the graph has $x = 1$ and $y = f(0) = ((0)^2+3)(5(0)+1)^{1/2} = 3$. Thus, $(x_0, y_0) = (0, 3)$.
- The slope of the tangent is given by $f'(x) = (x^2+3) \cdot \frac{1}{2}(5x+1)^{-1/2} \cdot 2 + (5x+1)^{1/2} \cdot 2x$. So at $x = 0$, we get slope $= f'(0) = ((0)^2+3) \cdot \frac{1}{2}(5(0)+1)^{-1/2} \cdot 5 + (2(0)+1)^{1/2} \cdot 2(0) = 3 \cdot \frac{1}{2} \cdot 5 + 0 = \frac{15}{2}$.
- The equation for the tangent line is

$$y = \frac{15}{2}(x-0) + 3.$$

Again, here is a picture of what we just found: