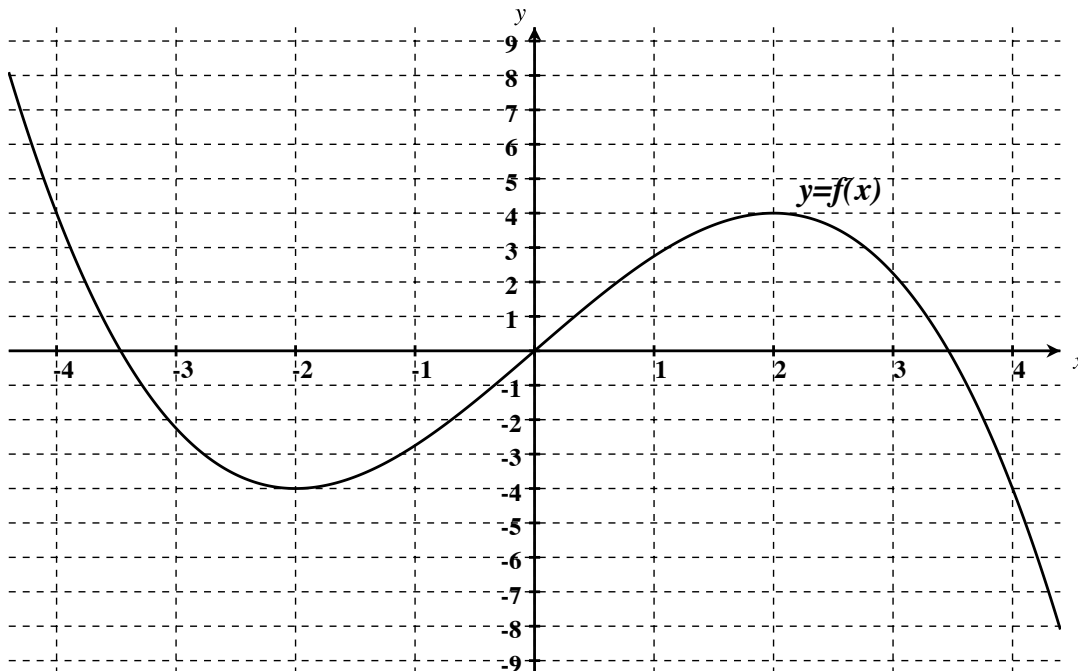


**Math 112**  
Group Activity: Graphs of Derivatives

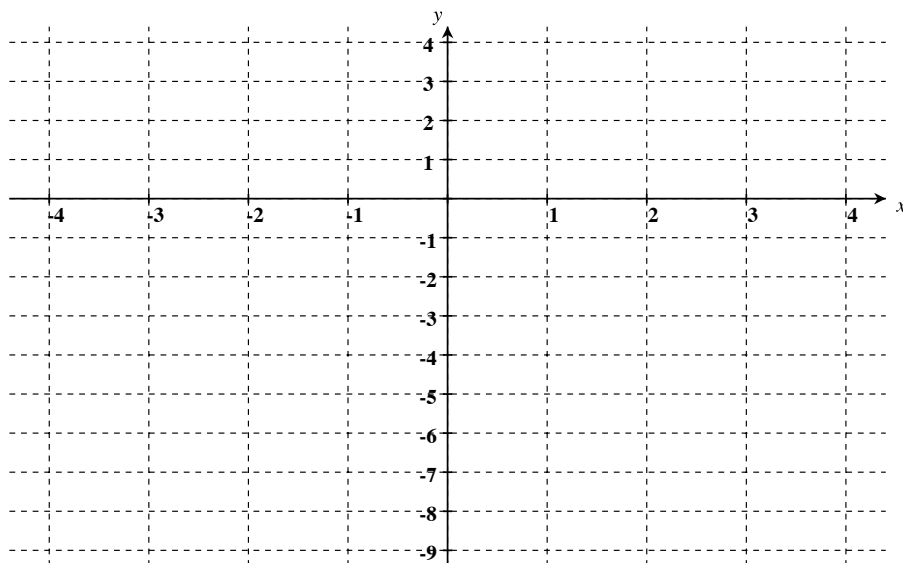
The graph below shows the function  $y = f(x)$ .



1. Recall that  $f'(a)$  gives the slope of the line tangent to  $f(x)$  at  $x = a$ .
  - (a) List all values of  $a$  at which  $f'(a) = 0$ .
  - (b) Give four values of  $a$  at which  $f'(a)$  is *negative*.
  - (c) Give three values of  $a$  at which  $f'(a)$  is *positive*.
  - (d) Describe how you can tell by looking at the graph of  $f(x)$  whether the value of  $f'(a)$  will be positive, negative, or zero.
  - (e) This graph has some nice symmetry. Without computing any slopes, use your ruler to verify that the following are true for this graph:  $f'(-1) = f'(1)$ ,  $f'(-3) = f'(3)$ , and  $f'(-4) = f'(4)$ .
  - (f) The following table contains some of the values of  $f'(x)$ . Fill in the remaining entries in the table. To fill in each entry, either use entries already in the table along with the symmetry described in part (e) or draw an appropriate tangent line and compute its slope.

$x$	-4	-3	-2	-1	0	1	2	3	4
$f'(x)$	-9			2.25				-3.75	

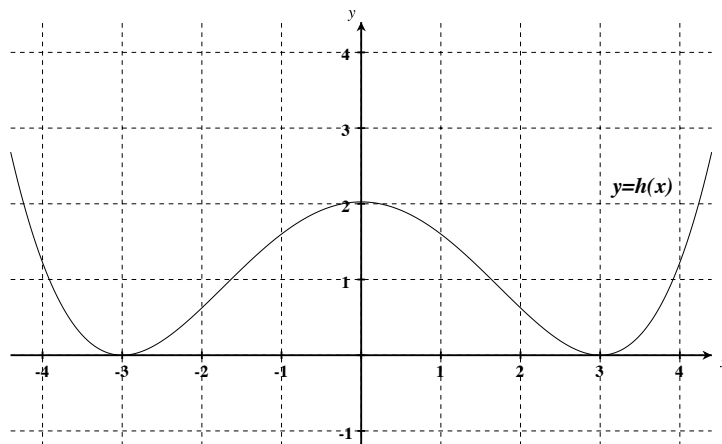
- (g) Use the values in the table to sketch the graph of  $f'(x)$  on the axes below. Notice that slopes of tangent lines to the graph of  $f(x)$  become  $y$ -values on the graph of  $f'(x)$ .



Notice that:

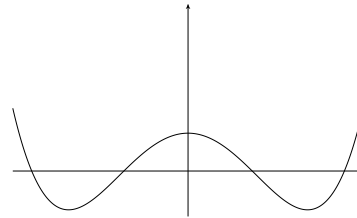
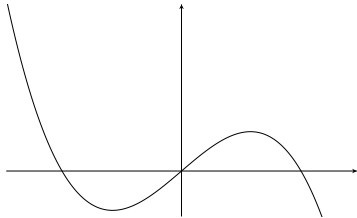
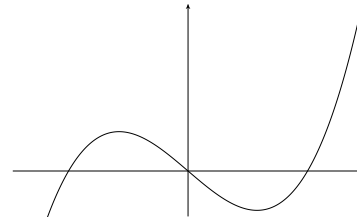
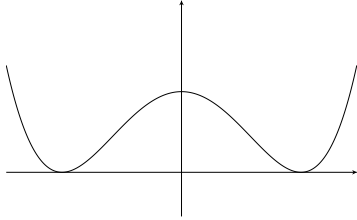
- The graph of  $f'(x)$  crosses the  $x$ -axis at those values of  $x$  where  $f(x)$  has horizontal tangent lines.
- The graph of  $f'(x)$  is *below* the  $x$ -axis (i.e., its  $y$ -values are *negative*) on the intervals where the graph of  $f(x)$  is *decreasing*.
- The graph of  $f'(x)$  is *above* the  $x$ -axis (i.e., its  $y$ -values are *positive*) on the intervals where the graph of  $f(x)$  is *increasing*.

2. Now consider the following graph of a function  $h(x)$ .

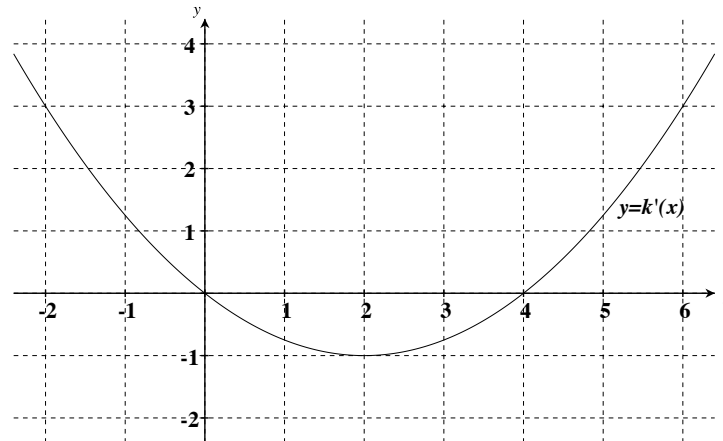


- List all values of  $x$  at which the graph of  $h'(x)$  crosses the  $x$ -axis.
- List all intervals on which the graph of  $h'(x)$  is *below* the  $x$ -axis.
- List all intervals on which the graph of  $h'(x)$  is *above* the  $x$ -axis.

(d) Which of the following is the graph of  $h'(x)$ ?



3. Below is the graph of  $k'(x)$ , the derivative of a function  $k(x)$ . The graph of  $k(x)$  is not shown.



(a) List all values of  $x$  at which the graph of  $k(x)$  has horizontal tangent lines.

(b) List all intervals on which the graph of  $k(x)$  is *increasing*.

(c) List all intervals on which the graph of  $k(x)$  is *decreasing*.

(d) Which of the following could be the graph of  $k(x)$  on the interval from  $x = 0$  to  $x = 4$ ?

