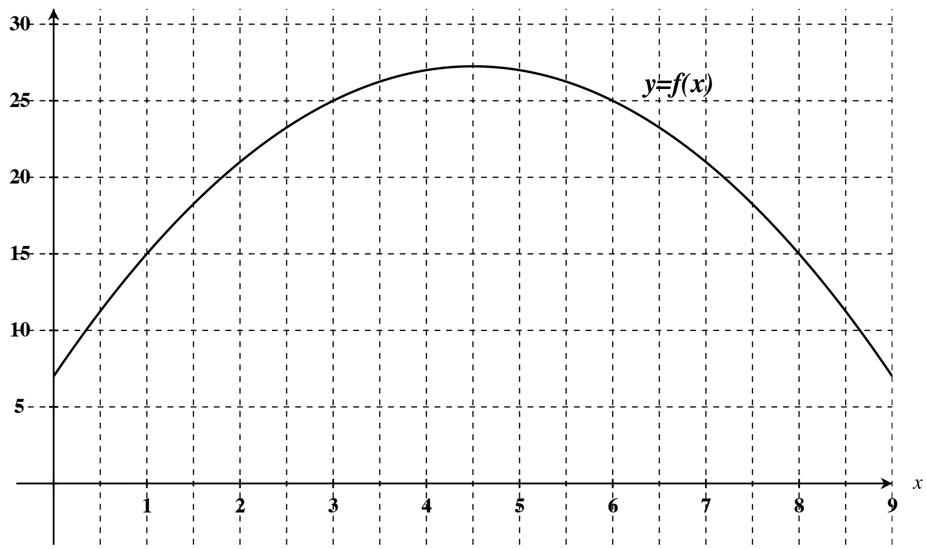
Name: ______________________ Section: __________

Math 112

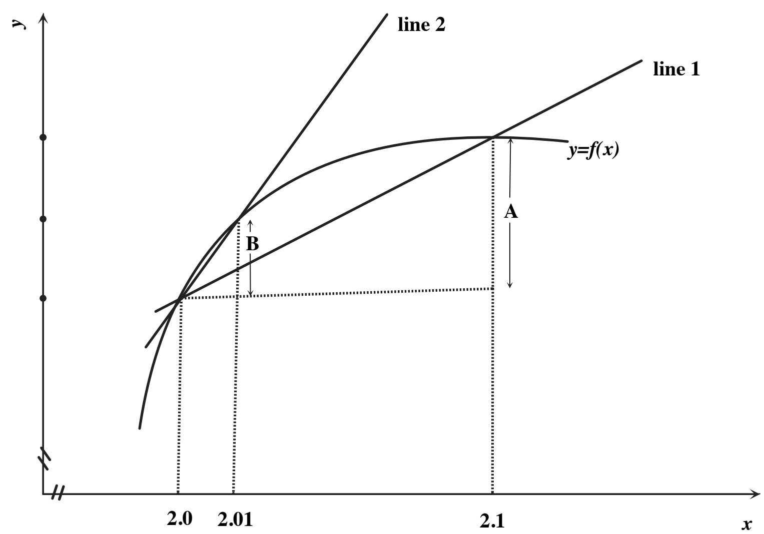
Group Activity: Tangents and Secants

The graph below is of a function $y = f(x)$.



1. Draw a tangent line to the graph of $f(x)$ at $x = 2$ and compute its slope. What is the functional notation for the slope you just computed?

2. Below is a magnification of the graph of $f(x)$ near $x = 2$. (It's not to scale and I've exaggerated the curviness of $f(x)$.) There are three points marked on the y-axis. Their heights are $f(2.0)$, $f(2.1)$ and $f(2.01)$. Label them correctly.



3. In terms of $f(2.0)$, $f(2.1)$ and $f(2.01)$, what are the values of the lengths marked A and B and the slopes of line 1 and line 2 in the picture above?

$A =$ slope of line 1 $=$

$B =$ slope of line 2 $=$

4. Which is closer to the slope of the tangent line to $f(x)$ at $x = 2$: the slope of line 1 or the slope of line 2?

5. How might you draw a line whose slope is even closer to the slope of the tangent to $f(x)$ at $x = 2$?

6. The formula for $f(x)$ is: $f(x) = -x^2 + 9x + 7$.

   (a) The slope of line 1 is given by $\dfrac{f(2.1) - f(2.0)}{0.1}$. Use the formula for $f(x)$ to compute the exact value of this slope.

   (b) Use the formula for $f(x)$ to compute the exact value of the slope of line 2.

   (c) Your work in parts (a) and (b) forms the first two lines of the following table. We've completed the next two lines for you. Use the pattern established in the table to complete the last two lines of the table. (Do not try to use your calculator to finish the table—just follow the pattern that you see emerging. Most calculators will round and the rounding error will cause the pattern stop.)

h	$2+h$	$f(2+h)$	f(2)	$f(2+h) - f(2)$	$\dfrac{f(2+h)-f(2)}{h}$
0.1	2.1	21.49	21	0.49	4.9
0.01	2.01	21.0499	21	0.0499	4.99
0.001	2.001	21.004999	21	0.004999	4.999
0.0001	2.0001	21.00049999	21	0.00049999	4.9999
0.00001					
0.000001					

   (d) Do you agree that the right-most column of the table gives slopes of secant lines that are getting closer and closer to the tangent line to $f(x)$ at $x = 2$? What would you predict is the exact value of $f'(2)$, the slope of the tangent line to $f(x)$ at $x = 2$? Is this close to the slope you computed using the graph in #1?

7. We'll use a similar method to compute $f'(3)$, the slope of the line tangent to $f(x)$ at $x = 3$, but this time we'll use algebra to make a smaller table. Again, the formula for $f(x)$ is:

$$f(x) = -x^2 + 9x + 7.$$

   (a) Compute and simplify the expression $\dfrac{f(3+h) - f(3)}{h}$.

   (b) What is the graphical interpretation of the expression in part (a)? (Your answer should begin "It is the slope of the...".)

   (c) Use your answer to part (a) to quickly fill in the following table:

h	$\dfrac{f(3+h)-f(3)}{h}$
0.1	
0.01	
0.001	
0.0001	

<— evaluate this using your answer to (a)

   (d) Do you agree that the right-most column of the table gives slopes of secant lines that are getting closer and closer to the tangent line to $f(x)$ at $x = 3$? What would you predict is the exact value of $f'(3)$, the slope of the tangent line to $f(x)$ at $x = 3$?

   (e) On the graph at the beginning of this activity, draw the line tangent to $f(x)$ at $x = 3$ and compute its slope. Is it close to the value you just found for $f'(3)$?

8. Finally, let's find a function that gives $f'(a)$, the slope of the tangent line to $f(x)$ at $x = a$. Again,

$$f(x) = -x^2 + 9x + 7.$$

(a) Compute and simplify the expression $\dfrac{f(a+h) - f(a)}{h}$.

(b) What is the graphical interpretation of the expression in part (a)? (Your answer should begin "It is the slope of the...".)

(c) If you take progressively smaller values for h, your answer to part (a) approaches some value that depends on a. What is that value, what is its graphical interpretation, and how would you use functional notation to express it?

(d) Use your answer to part (c) to compute $f'(2)$ and $f'(3)$. Do these match your previous computations?

(e) Use your answer to part (c) to compute $f'(1)$ and $f'(5)$.