1. Rates of Change for some function \( f(t) \)

- \( \frac{f(b) - f(a)}{b - a} = \text{"Rate of Change from } t = a \text{ to } t = b\" \)
- \( \text{the slope of the secant line through the graph at } t = a \text{ and } t = b\". 
  - If \( a = 0 \), then we call this the overall rate of change. Otherwise we call it the average rate of change over an interval.
- The equation \( \frac{f(t)}{t} = \text{"The slope of the diagonal line through the graph of } f(t)\". 

2. Business Terms and Techniques

- **Basic Functions Relating to Business**: For these functions, you plug in a quantity, \( q \), and the function outputs a dollar amount which is related to the amount of money in, the amount of money out, or both.
  - \( TR = TR(q) = pq = \text{total revenue for selling } q \text{ Things (money brought in before cost)} \)
  - \( TC = TC(q) = VC(q) + FC = \text{total cost for selling } q \text{ Things.} \)
    - \( FC = TC(0) = \text{fixed cost = } 'y'-\text{intercept of the } TC \text{ graph.} \)
    - \( VC = VC(q) = \text{variable cost = cost not including the fixed cost.} \)
  - \( P = P(q) = \text{Profit} = TR(q) - TC(q). \)

- **Average Business Functions**: For these functions, you plug in a quantity, \( q \), and the function outputs the dollars per item.
  - \( AR = AR(q) = \frac{TR(q)}{q} = \text{average revenue = price per item when selling } q \text{ items.} \)
  - \( AC = AC(q) = \frac{TC(q)}{q} = \text{average cost = cost per item when producing } q \text{ items.} \)
  - \( AVC = AVC(q) = \frac{AVC(q)}{q} = \text{average variable cost = variable cost per item when producing } q \text{ items.} \)

- **Marginal Functions**: For these functions, you plug in a quantity, \( q \), and the function outputs the dollar amount that comes about when the quantity increases by ‘one unit’.
  - \( MR = MR(q) = TR(q + \text{‘one unit’}) - TR(q) = \text{"change in revenue when selling one more item after } q\" \)
  - \( MC = MC(q) = TC(q + \text{‘one unit’}) - TC(q) = \text{"change in cost when selling one more item after } q\" \)

- **Maximum Profit**: We have many ways to find the maximum profit. Here are a few:
  - **Graphically**
    - Find the quantity where the \( TR \) graph is above the \( TC \) graph by the greatest difference.
    - Find the quantity where the slope of the \( TR \) graph is the same as the slope of the \( TC \) graph.
  - **Symbolically**
    - Solve for \( q \) in the equation \( MR(q) = MC(q) \).
    - If \( P(q) = TR(q) - TC(q) \) is a quadratic function, then we can use the vertex formula.
3. **Quadratic Functions**

- A quadratic function is any expression of the form $ax^2 + bx + c$ with $a \neq 0$.
  - If $a < 0$, then the parabola opens downward (it is frowning).
  - If $a > 0$, then the parabola opens upward (it is smiling).

- To find $x$ coordinate of a vertex, we use the vertex formula: $x = -\frac{b}{2a}$. To get the maximum/minimum value of the function, we then plug this value back into the function. The vertex is a maximum if the parabola opens downward and a minimum if the parabola opens upward.

- To solve an equation involving a quadratic, get everything to one side of the equation so that it looks like: $ax^2 + bx + c = 0$ and use the quadratic formula (if you are entering it into your calculator remember to put parentheses where I have indicated):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{(2a)}.$$