MATH 112
Exam I
Winter 2017

Name _____

Student ID #_____ Section _____

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:_____

| | | |
|---|---|---|
| 1 | 12 | |
| 2 | 20 | |
| 3 | 9 | |
| 4 | 9 | |
| Total | 50 | |

- Check that your exam contains 4 problems.
- You are allowed to use a TI-30XIIS calculator, a ruler, and one sheet of hand-written notes. All other sources are forbidden.
- Do not use scratch paper. If you need more room, use the back of the page and indicate to the grader you have done so.
- Turn your cell phone OFF and put it away for the duration of the exam.
- You may not listen to headphones or earbuds during the exam.
- You must show your work. Clearly label lines and points that you are using and show all calculations. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- When rounding is necessary, you may round your final answer to two digits after the decimal.
- There are multiple versions of the exam, you have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.
- Put your name on your sheet of notes and turn it in with the exam.

GOOD LUCK!

1. (12 points) Compute the indicated derivative. DO NOT SIMPLIFY. Put a box around your answer.

   (a) $f(x) = (5 - 7x^3) \cdot (\sqrt[3]{x^4 + 2x})$

   $f'(x) =$

   (b) $y = \left(\dfrac{x^5 + 1}{2 - 3x^2}\right)^8$

   $\dfrac{dy}{dx} =$

   (c) $D(t) = t^3 - \sqrt{t} - \dfrac{4}{t}$

   $D'(t) =$

   $D''(t) =$

2. (20 points) The demand function for a product is given by $p = 147.4 - x$, where $x$ is the number of units and $p$ is the **price per unit**, in dollars.

   (a) Compute the **total revenue** collected from the sale of 50 units.

   ANSWER: \$_____

   (b) Find the production level at which **marginal revenue** is \$101.40 per unit.

   ANSWER: $x =$_____units

   (c) Give the longest interval on which **total revenue** is increasing.

   ANSWER: from $x =$_____to $x =$_____units

   For the same product, the **cost per unit** (i.e., average cost) is $\dfrac{800}{x} + 1 + 0.2x$ dollars.

   (d) Find formulas for **total cost** and **marginal cost**.

   ANSWER: $TC(x) =$_____

   $MC(x) =$_____

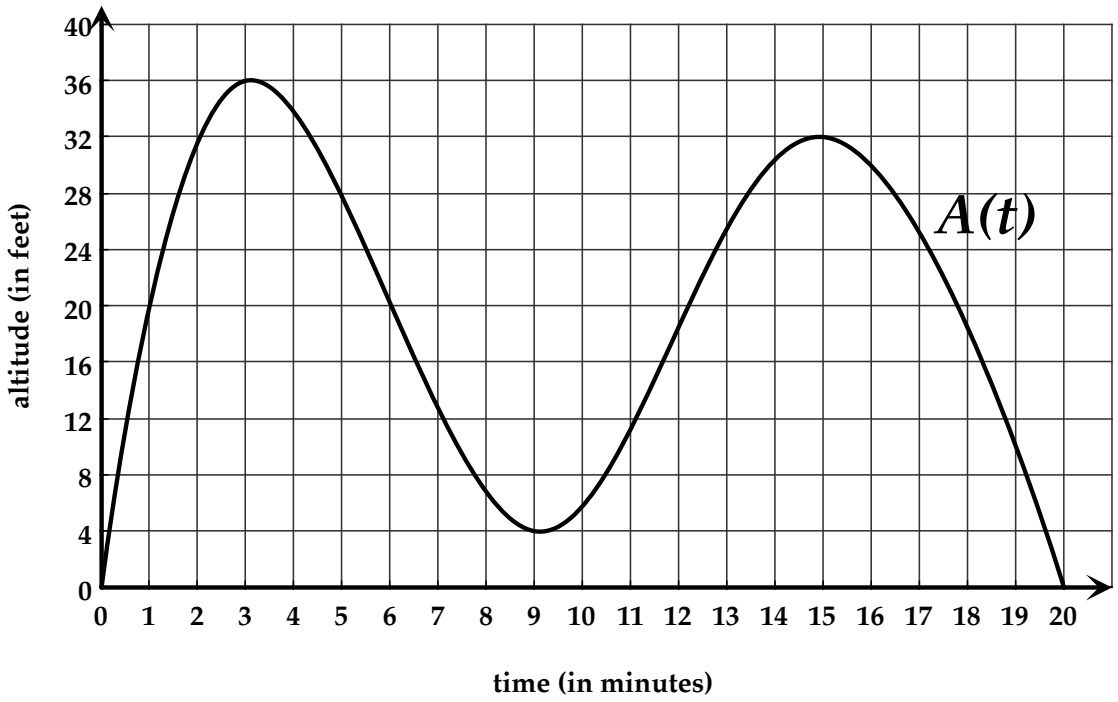
   (e) Which costs more to produce: the $501^{st}$ unit or the $701^{st}$ unit? (Show your work.)

   ANSWER: (circle one) the $501^{st}$ the $701^{st}$

   (f) Find the production level at which **profit** is maximized.

   ANSWER: $x =$_____units

3. (9 points) The graph below shows $A(t)$, the **altitude** of a balloon as it rises and falls for twenty minutes.




(a) Put in order, from smallest to largest, the following quantities.
(You do not have to compute these values or show any work. Just put them in order.)

**i.** the balloon's instantaneous rate of ascent at $t = 10$

**ii.** the balloon's average rate of ascent from $t = 1$ to $t = 5$

**iii.** $A'(2)$

**iv.** $\dfrac{A(2.01) - A(2)}{0.01}$

ANSWER: (smallest) _____ _____ _____ _____ (largest)

(b) List all the times at which the graph of the balloon's instantaneous rate of ascent crosses the $t$-axis.

ANSWER: (list all) $t =$_____minutes

(c) List the intervals on which $A'(t)$, the derivative of altitude, is negative.

ANSWER: from $t =$_____to $t =$_____minutes

from $t =$_____to $t =$_____minutes

4. (9 points) Two remotely controlled cars travel on a long straight track. The **red car's distance traveled** is given by

$$R(t) = 5t - 0.5t^2,$$

where time is in minutes and distance is in feet.

The **green car's distance traveled** is given by a function $G(t)$ whose formula we do not know explicitly. But we do know that

$$G(a+h) - G(a) = 6h - 2ah - h^2.$$

(a) Which car travels farthest in the first three minutes (from $t = 0$ to $t = 3$)? (As always, show your work.)

ANSWER: (circle one) red car green car

(b) Find the formula for $G'(t)$.

ANSWER: $G'(t) =$_____

(c) Find all times at which the red and green cars have the same instantaneous speed.

ANSWER: (list all) $t =$_____minutes